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CONTROL OF DELAY SYSTEMS – A MEROMORPHIC FUNCTION APPROACH

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Abstract: Systems with delay terms at the left (and the right) side of differential equations are addressed. Analysis and synthesis of delay systems can be conveniently studied through a special ring of RQ-meromorphic functions. The control methodology is based on the solution of Diophantine equations in this ring. Final controllers result in the Smith predictor like structure. A scalar parameter is defined as a „tuning knob" for controller parameters and control behaviour. A simple pole-assignment-like tuning idea is utilized. First and second order cases are derived for illustration and simulation.

Keywords: Rings, algebraic tools, Diophantine equation, parameterization, Smith predictor.

1 INTRODUCTION

It is a well-known fact that many real-life systems and processes are afflicted by delays which are traditionally considered and modelled in input-output manner. However, inner dynamic loops of a system can naturally be delayed as well, which gives rise to the notion of an internal delay. Such systems are also called hereditary or anisochronic, see e.g. (Kamen 1975, Zítek 1986), due to the fact that the system state is a function of a time segment rather than a single time instant, and they have many interesting features (e. g. an infinite spectrum). The presence of both delays, i.e. output and internal one, entails problems with controllers design due to the significant impression to feedback properties of a control system, such as stability and periodicity.

This paper suggests a simple and easy-to-proceed algebraic controller design for stable systems with both (lumped) delays avoiding any time delay approximation. A controlled system is described by a transfer function which is a ratio of two terms in a special algebraic ring - A ring of stable and proper retarded quasipolynomial meromorphic functions ($\mathbf{R}_{MS}$), see e.g. (Zítek and Kučera 2003, Pekař and Prokop 2009). A term of this ring is a ratio of two quasipolynomials (El'sgol'ts and Norkin 1973) where the denominator quasipolynomial is stable and the whole ratio is proper with respect to the highest s-power. The Bézout identity along with the Youla-Kučera parameterization (Kučera 1993) is utilized and final controllers ensure feedback loop stability, tracking of the step reference and load disturbance attenuation. Herein used control system represents a simple 1DOF (degree-of-freedom) control structure.

Final obtained anisochronic controllers structures own an optional scalar parameter (or parameters) which allows a controller to be appropriately tuned. To name but a few, pole-assignment (shifting) methodology was presented in (Michiels et al. 2002, Vyhlídal 2003), modified equalization method can be found e.g. in (Pekař and Prokop 2008) and an application of the gain margin principle (the Nyquist criterion) was suggested in (Zítek and Víteček 1999, Pekař and Prokop 2007). In this contribution, very simple and intuitive pole-placement-like tuning idea is presented since tuning is not the main goal here. Simulation examples verify and demonstrate the usability of the method presented in this paper.

2 RMS RING

Algebraic tools such as rings and linear equations are frequently preferred in modern control theory. Different rings require various approximations of delay terms which reduce quality of a model. The most known is the Padè approximation, respecting the relative degree of the original transfer function. As a consecution, we loose plant dynamics information and final control design obviously gives controllers of quite high degrees. This paper utilizes a ring of stable and proper meromorphic functions $\mathbf{R}_{MS}$ developed especially for delay systems and omitting any approximation.

An element of this ring is a ratio of two retarded quasipolynomials $y(s)/x(s)$ where a retarded quasipolynomial $x(s)$ of degree n means

$$x(s) = s^n + \sum_{i=0}^{n-1}\sum_{j=1}^{h} x_{ij} s^i \exp(-\vartheta_{ij} s),\ \vartheta_{ij} \geq 0 \tag{1}$$

Note that the highest s-power in a retarded quasipolynomial is not affected by exponentials. A more general notion called neutral quasipolynomials also can be used in this sense, see details in (Pekař and Prokop 2009) where some discrepancies about this original definition are discussed as well. The denominator of the ratio in $\mathbf{R}_{MS}$ is supposed to be stable, while the numerator $y(s)$ of an element in $\mathbf{R}_{MS}$ can be factorized in the form $y(s) = \tilde{y}(s)\exp(-\tau s)$, where $\tau \geq 0$ and $\tilde{y}(s)$ is any retarded quasipolynomial. Quasipolynomial (1) is stable when it owns no finite zero s_0 such that $\mathrm{Re}\{s_0\} \geq 0$. For stability tests, see e.g. in (Zítek and Víteček 1999). The ratio $y(s)/x(s)$ is called proper when the degree of the numerator is less or equal to the degree of the denominator.

The $\mathbf{R}_{MS}$ ring is now used for a model description where the transfer function of a plant or a controller is expressed as a ratio of two terms of the ring. One can consider it as a coprime factorization of a transfer function in the form of a ratio of two quasipolynomials. Hence, this contribution deals with a first order and a second order delayed plants, respectively, which are described as

$$G(s) = \frac{b\exp(-\tau s)}{s + a\exp(-\vartheta s)} = \frac{\dfrac{b\exp(-\tau s)}{m(s)}}{\dfrac{s + a\exp(-\vartheta s)}{m(s)}} = \frac{B(s)}{A(s)} \tag{2}$$
$$A(s), B(s) \in \mathbf{R}_{MS};\ \tau, \vartheta \geq 0$$

and

$$G(s) = \frac{b\exp(-\tau s)}{(s + a_1\exp(-\vartheta s))(s + a_2)} = \frac{\dfrac{b\exp(-\tau s)}{m(s)}}{\dfrac{(s + a_1\exp(-\vartheta s))(s + a_2)}{m(s)}} = \frac{B(s)}{A(s)} \tag{3}$$
$$A(s), B(s) \in \mathbf{R}_{MS};\ \tau, \vartheta \geq 0$$

respectively, where $m(s)$ is an appropriate stable (quasi)polynomial of the first and second order, respectively (due to the coprimeness). A suitable form of $m(s)$ is contentious and depends on user's requirements. System (2) is stable iff

$$a\vartheta \in (0, \pi/2) \tag{4}$$

see (Górecki et al. 1989), similarly for (3).

3 CONTROL DESIGN IN GENERAL

Let us briefly describe the principle of algebraic controller design in the $\mathbf{R}_{MS}$ ring. Consider a simple feedback control system structure as in Fig.1.

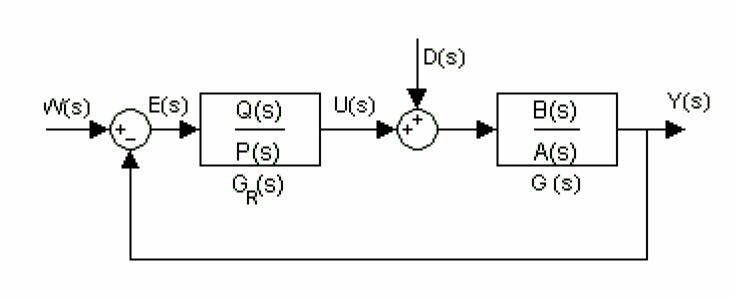

Figure 1: Simple feedback control loop

In the figure, $W(s)$ is the Laplace transform of the reference signal, $D(s)$ is that of the load disturbance, $E(s)$ is transformed control error, $U(s)$ represents the plant input, and $Y(s)$ is the plant output (controlled value) in the Laplace transform. The plant transfer function is depicted as $G(s)$, and $G_R(s)$ stands for a controller in the scheme.

Control system inputs have forms

$$W(s)=\frac{H_W(s)}{F_W(s)},\ D(s)=\frac{H_D(s)}{F_D(s)} \tag{5}$$

where $H_W(s),\ H_D(s),\ F_W(s),\ F_D(s)\ \in \mathbf{R}_{MS}$.

The basic requirements on the control systems are closed-loop stability, asymptotical reference tracking and load disturbance attenuation.

Consider that $A(s), B(s) \in \mathbf{R}_{MS}$ are coprime. It is a well known fact, see e.g. (Kučera 1993, Zítek and Kučera 2003), that the closed-loop system stability is ensured by the solution $P(s), Q(s) \in \mathbf{R}_{MS}$ (if it exists) of the Bézout identity

$$A(s)P(s)+B(s)Q(s)=1 \tag{6}$$

A particular stabilizing solution, $P_0(s), Q_0(s)$, can be then parameterized as

$$\begin{aligned} P(s)&=P_0(s)-B(s)Z(s)\\ Q(s)&=Q_0(s)+A(s)Z(s);\ Z(s)\in \mathbf{R}_{MS} \end{aligned} \tag{7}$$

This parameterization is used to fulfill other control and performance requirements. Since coprime factorization always exists for stable plants (Pekař and Prokop 2009), a solution of (6) exists as well.

Parameterization (7) via the choice of $Z(s)$ enables to find the solution of (6), so that the requirement of asymptotic disturbance rejection is accomplished. This control condition is assured iff $F_D(s)$ divides $P(s)$, i.e. all unstable zeros of $F_D(s)$ are those of $P(s)$. Notice that quasipolynomials yet offer more variations than polynomials, how to solve this task.

Asymptotic reference tracking is also one of the most basic control conditions. Similarly as for disturbance rejection, reference tracking requires $P(s)$ to be divisible by $F_W(s)$.

4 CONTROL OF THE FIRST ORDER PLANT

The derivation of controllers' structures for a stable and an unstable plant of transfer function (2) using the $\mathbf{R}_{MS}$ ring is demonstrated in this section. The task will be independently solved for a stable and an unstable plant; recall that stability condition is given by (4).

Stable case

Consider a plant (2) satisfying (4) where $m(s)$ is chosen (for the simplicity reason) as polynomial $s+m_0$. Control system inputs, *W(s)* and *D(s)*, can have naturally various structures; however, assume the simplest practical case that both external inputs are from the class of step functions, hence

$$W(s)=\frac{H_W(s)}{F_W(s)}=\frac{\frac{w_0}{m_w(s)}}{\frac{s}{m_w(s)}},\ D(s)=\frac{H_D(s)}{F_D(s)}=\frac{\frac{d_0}{m_d(s)}}{\frac{s}{m_d(s)}} \tag{8}$$

where $m_w(s)$ and $m_d(s)$ are arbitrary stable (quasi)polynomials of degree one, say for the simplicity, $s+m_0$ again.

Solve (6) by the choice $Q_0=1$ yielding

$$P_0(s)=\frac{s+m_0-b\exp(-\tau s)}{s+a\exp(-\vartheta s)} \tag{9}$$

Recall that this particular solution internally stabilizes the control system.

Now parameterize the solution according to (7) to obtain controllers asymptotically rejecting the disturbance

$$P(s)=\frac{s+m_0-b\exp(-\tau s)}{s+a\exp(-\vartheta s)}-\frac{b\exp(-\tau s)}{s+m_0}Z(s) \tag{10}$$

The numerator of *P*(*s*) has to have at least one zero root. Moreover, it is appropriate to have *P*(*s*) in a simple form, which is fulfilled e.g. when

$$Z(s)=\left(\frac{m_0}{b}-1\right)\frac{s+m_0}{s+a\exp(-\vartheta s)} \tag{11}$$

providing

$$P(s)=\frac{s+m_0(1-\exp(-\tau s))}{s+a\exp(-\vartheta s)},\ Q=\frac{m_0}{b} \tag{12}$$

Thus, final controller's structure is the following

$$G_R(s)=\frac{m_0[s+a\exp(-\vartheta s)]}{b[s+m_0(1-\exp(-\tau s))]} \tag{13}$$

Note that the controllers are of the anisochronic type because of a delay in the transfer function denominator. It is naturally possible to take $m(s)$ as a quasipolynomial instead of polynomial; however, this option would make a controller more complicated. The importance of $m(s)$ reveals from the closed loop transfer function

$$G_{WY}(s)=\frac{Y(s)}{W(s)}=\frac{m_0\exp(-\tau s)}{s+m_0} \tag{14}$$

i.e. $m(s)$ appears as a characteristic (quasi)polynomial of the closed loop.

The obtained control structure can be easily compared with the well-known Smith predictor structure, see e.g. in (Pekař and Prokop 2008).

Unstable case

When the plant (2) is unstable, a suitable choice of $m(s)$ is more complicated. If one takes $m(s)$ as in the previous subsection, the solution of (6) would be excessively complicated, due to the stability and properness of the solution (ring conditions). To avoid this problem, do the coprime factorization as follows

$$G(s)=\frac{b\exp(-\tau s)}{s+a\exp(-\vartheta s)}=\frac{\dfrac{b\exp(-\tau s)}{s+a\exp(-\vartheta s)+qb\exp(-\tau s)}}{\dfrac{s+a\exp(-\vartheta s)}{s+a\exp(-\vartheta s)+qb\exp(-\tau s)}}=\frac{B(s)}{A(s)} \tag{15}$$

where q is a real number. Stability analysis via the Michailov stability criterion of the common denominator can be found in (Pekař and Prokop 2007), thus, it is stable if

$$-\frac{a}{b}<q<\frac{\omega_C-a\sin(\vartheta\omega_C)}{b\sin(\tau\omega_C)} \tag{16}$$

where ω_C is given by the solution of

$$\omega_C=a\frac{-\sin[(\vartheta-\tau)\omega_C]}{\cos(\tau\omega_C)}\neq 0 \tag{17}$$

Obviously, stabilizing equation gives the solution

$$Q_0=q, P_0=1 \tag{18}$$

Parameter q can be viewed as a proportional controller in the inner feedback loop and the common quasipolynomial is the characteristic quasipolynomial of the loop. Taking

$$Z(s)=\frac{m_0}{b}\frac{s+a\exp(-\vartheta s)+qb\exp(-\tau s)}{s+m_0},\ m_0>0 \tag{19}$$

the following controller structure satisfying asymptotic reference tracking and disturbance rejection is obtained

$$G_R(s)=\frac{1}{b}\frac{(qb+m_0)s+qbm_0+am_0\exp(-\vartheta s)}{s+m_0(1-\exp(-\tau s))} \tag{20}$$

Again, a controller of an anisochronic type is obtained, yet more complex in comparison with the preceding one. The closed loop transfer function then yields

$$G_{WY}(s)=\frac{Y(s)}{W(s)}=\frac{(qb+m_0)s+qbm_0+am_0\exp(-\vartheta s)}{(s+m_0)(s+a\exp(-\vartheta s)+qb\exp(-\tau s))}\exp(-\tau s) \tag{21}$$

The final controller thus has two degrees of freedom (q, m_0). It is naturally possible to take $q = m_0$ to reduce the degrees of freedom and to simplify the tuning process.

5 CONTROL OF THE SECOND ORDER PLANT

The demonstrated algebraic control design is now utilized to model (3) – a stable case. Factorize the plant as

$$G(s)=\frac{b\exp(-\tau s)}{(s+a_1\exp(-\vartheta s))(s+a_2)}=\frac{\dfrac{b\exp(-\tau s)}{(s+m_0)^2}}{\dfrac{(s+a_1\exp(-\vartheta s))(s+a_2)}{(s+m_0)^2}}=\frac{B(s)}{A(s)},\ m_0>0 \qquad (22)$$

thus the common denominator in both terms in the $\mathbf{R}_{MS}$ ring has a double real zero. Obviously, one can choose another one stable (quasi)polynomial. Recall that this (quasi)polynomial then appears as a factor of the characteristic (quasi)polynomial of the closed loop.

A stabilizing controller as a particular solution of (6) is e.g.

$$Q_0=1, P_0(s)=\frac{(s+m_0)^2-b\exp(-\tau s)}{(s+a_1\exp(-\vartheta s))(s+a_2)} \qquad (23)$$

Taking

$$Z(s)=\left(\frac{m_0^2}{b}-1\right)\frac{(s+m_0)^2}{(s+a_1\exp(-\vartheta s))(s+a_2)} \qquad (24)$$

in the parameterization (7) it is obtained a class of controllers in a quite simple form satisfying both additional conditions, with the transfer function

$$G_R(s)=\frac{m_0^2}{b}\frac{(s+a_1\exp(-\vartheta s))(s+a_2)}{s^2+2m_0 s+m_0^2(1-\exp(-\tau s))} \qquad (25)$$

Note the closed loop transfer function then reads

$$G_{WY}(s)=\frac{Y(s)}{W(s)}=\frac{m_0^2\exp(-\tau s)}{(s+m_0)^2} \qquad (26)$$

6 CONTROLLERS TUNING

The suitable choice of m_0 and q can be an effective and simple tuning tool since these real numbers strongly influence all controller parameters and the characteristic (quasi)polynomial of the closed loop. The question of the right and/or optimal choice of m_0 has not been solved yet, although many possible attempts have been studied.

In this contribution a simple pole-placement-like method is applied. Notice that all closed loop characteristic equations (14), (21), (26) contain selectable parameters m_0 and/or q. Moreover, the closed loop poles can be placed directly for (14) and (26), whereas denominator of (21) is a product of two factors. One of them allows placing a pole directly however the second one is a quasipolynomial and thus one can not be sure that the chosen pole is dominant. Therefore, in simulation examples below, gain margin principle is utilized for q.

7 SIMULATION EXAMPLES

The aim of this chapter is to demonstrate the usability of the proposed algebraic method on some simulation examples.

Stable first order plant

Let the stable controlled system be described by the model

$$G(s)=\frac{0.6\exp(-4s)}{s+0.2\exp(-0.8s)}=\frac{\dfrac{0.6\exp(-4s)}{s+m_0}}{\dfrac{s+0.2\exp(-0.8s)}{s+m_0}} \tag{27}$$

Then the controller according to (13) reads

$$G_R(s)=\frac{m_0[s+0.2\exp(-0.8s)]}{6[s+m_0(1-\exp(-4s))]} \tag{28}$$

Try to place three different poles -0.2, -0.5, -1, i.e. m_0 = 0.2, m_0 = 0.5, m_0 = 1, and compare the simulation results ($u(t), y(t)$) in Fig.2. Reference signal is $w(t)$ = 1 for $t \in [0,50)$ and $w(t)$ = 2 for $t \in [50,150]$. Step input disturbance $d(t)$ = -0.1 enters at t = 100.

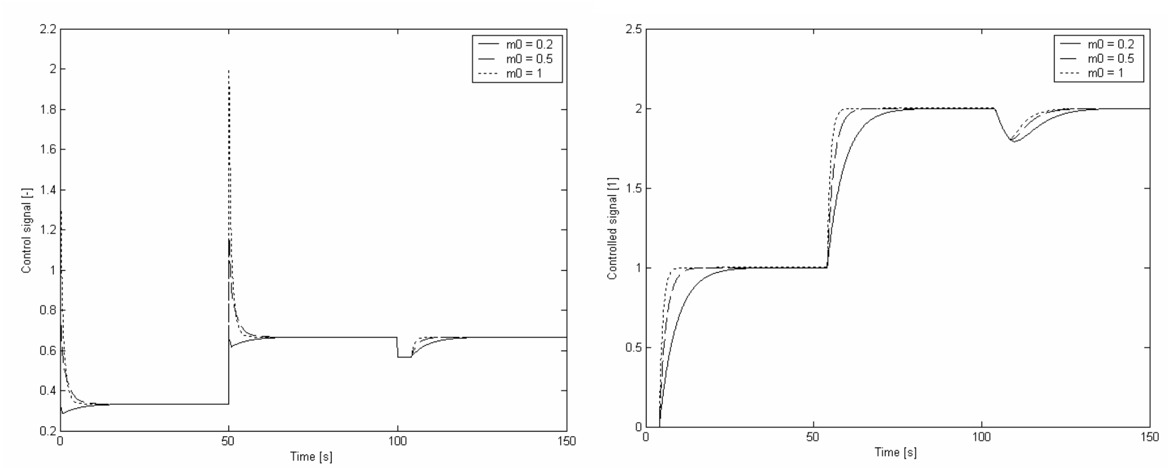

Figure 2: Closed-loop step responses for plant (27) and controller (28)

Unstable first order plant

Consider the unstable plant

$$G(s)=\frac{0.6\exp(-4s)}{s-0.2\exp(-0.8s)}=\frac{\dfrac{0.6\exp(-4s)}{s-0.2\exp(-0.8s)+q0.6\exp(-4s)}}{\dfrac{s-0.2\exp(-0.8s)}{s-0.2\exp(-0.8s)+q0.6\exp(-4s)}} \tag{29}$$

Algebraic control design (15)-(19) yields the controller

$$G_R(s)=\frac{5}{3}\frac{(0.6q+m_0)s+0.6qm_0-0.2m_0\exp(-0.8s)}{s+m_0(1-\exp(-4s))} \tag{30}$$

Conditions (16), (17) limit q as

$$\frac{1}{3}<q<0.564 \tag{31}$$

Imagine a proportional controller q and choose the gain margin A_m = 1.3. This requirement leads to q = 0.434, see details in (Pekař and Prokop 2007). The second selectable parameter choose as m_0 = 0.5, m_0 = 2, m_0 = 10. Simulation results are displayed in Fig.3. Reference signal is $w(t) = 1$ for $t \in [0,150)$ and $w(t) = 2$ for $t \in [150, 400]$. Step input disturbance $d(t) = -0.1$ enters at $t = 300$.

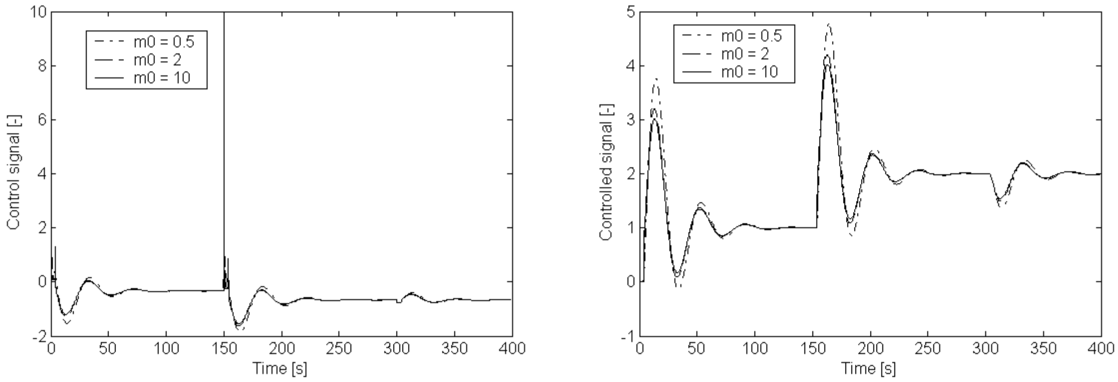

Figure 3: Closed-loop step responses for plant (29) and controller (30)

Control responses are obviously not very satisfactory. Thus, one can try to set selectable parameters more precisely or to utilize other control system structures (Pekař and Prokop 2007).

Stable second order plant

Let the second order stable plant be described as

$$G(s) = \frac{0.3\exp(-4s)}{(s+0.2\exp(-0.8s))(s+0.5)} = \frac{\dfrac{0.6\exp(-4s)}{(s+m_0)^2}}{\dfrac{(s+0.2\exp(-0.8s))(s+0.5)}{(s+m_0)^2}} \tag{32}$$

The corresponding controller (25) is then

$$G_R(s) = \frac{m_0^2}{0.3}\frac{(s+0.2\exp(-0.8s))(s+0.5)}{s^2+2m_0 s+m_0^2(1-\exp(-4s))} \tag{33}$$

The reference signal, the load disturbance and closed loop poles are chosen as for the first order stable case. Closed loop step responses are pictured in Fig.4.

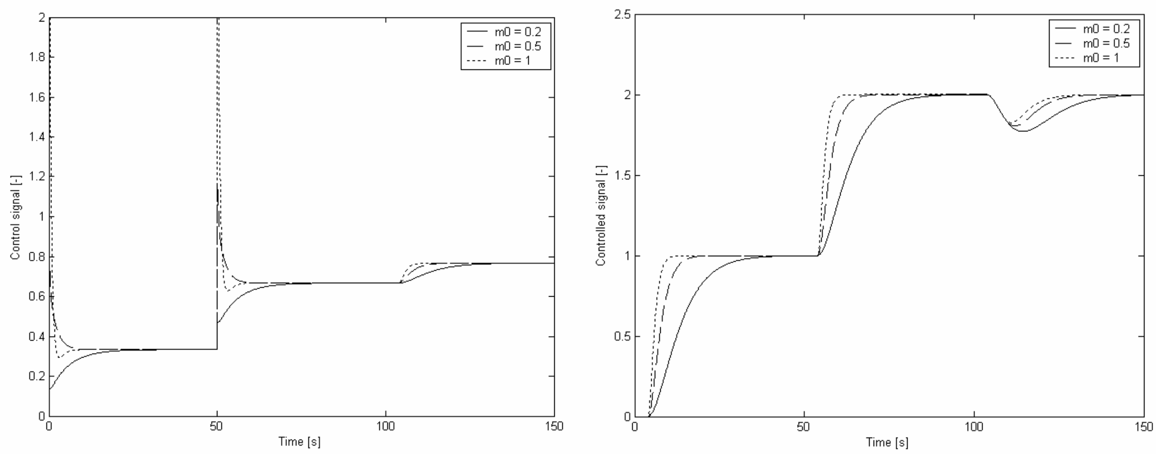

Figure 4: Closed-loop step responses for plant (32) and controller (33)

8 CONCLUSION

The problem of algebraic control design for stable and unstable processes with both input-output and internal has been solved in this contribution. These models are suitable e.g. for modelling of conventional high order systems. The proposed algebraic method for system description and control in the $\mathbf{R}_{MS}$ (stable and proper RQ meromorphic functions) ring does not involve the delay approximation. The controller structure is derived through the solution of the Bézout equation with the Youla-Kučera parameterization. The methodology enables to find various controllers that satisfy requirements on closed loop stability, (step) reference tracking and (step) load disturbance attenuation. The control system was chosen as the conventional 1DOF scheme. The final controllers can be tuned by one or two selectable parameters; among many possible tuning methods, a pole-placement-like idea has been adopted in this paper. The efficiency of the proposed method is verified on simulation examples.

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