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A NEW METHOD IN RELIABILITY-COST OPTIMISATION FOR SERIES-PARALLEL ELECTRICAL POWER SYSTEM

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Abstract

This paper addresses a new method of generalized genarating sequences (GGS) in reliability-cost optimization problem. The GGS is an extension based on some modifications of the standard universal genarating function (UGF). This method is convenient for computerized calculation of enumeration combinatorial problems thst arise in discrete optimization problem as encountred in parallel-series power systems. This method will be convenient to the complex combinatorial optimization problem as described below in the context of optimal redundancy-investment allocation problems (PRIOP). The GGS is used in bi-objective optimization functions PRIOP.

Keywords: Enumeration, Optimization, PRIOP, UGF, Bi-objective function, GGS.

1 INTRODUCTION AND PRELIMINARIES

Many complex problems need a fast method to be solved in short time, all the reliability combinatorial problems based on the classical reliability methods are the cases. Solution of various problems of discrete optimization leads to complex enumerating states procedures. The method of generalized generating sequences (GGS) was developed on computer based solution of such problems as in [1] and [2]. The method was used for solution of optimal redundancy problems in [3] and [4], for multi-state systems analysis as discussed in [5] and [6] and for power system reliability analysis in [7]. A comparison between this new method used and standard universal generating function for a complex system will be implemented.

The rest of this paper is outlined as follows: We start in Section 2 with the general polynomial form of generalized generating sequence. Next, a description of availability estimation based on UGF modified to GGS for redundancy-investment optimization problem in Section 3. A GGS algorithm is presented in section 4. In Section 5, we present the illustrative example. Conclusion is drawn in Section 6.

2 GENERAL POLYNOMIAL FORM OF GENERALISED GENERATING SEQUENCE

For the sake of clarity, we begin with a polynomial representation of the GGS, i.e. the GGS method will be first described in terms of common generating function used in combinatorial analysis problem. Let us begin with a simple example.

Example1. Consider a series system consisting of two units (Fig. 1). Remind that in reliability theory a system is called series if a failure of at least one unit leads to the system failure.

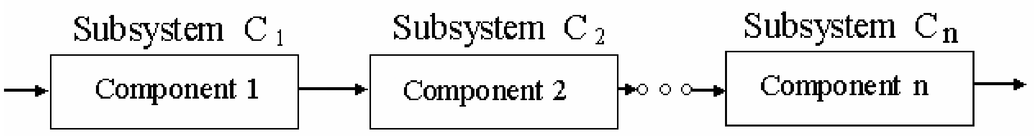

Figure 1: Series power units system

Let the first unit be characterized by the probability of successful operation $r_1 = 0.9$ and cost $c_1 = 1$, where as the second unit has, respectively, $r_2 = 0.8$ and $c_2 = 3$ and $r_n = 0.88$ and $c_n = 2$. Reliability of each unit can be improved by the use of hot redundancy (Fig. 2), which means that all units of the redundant group are in the same operation condition [8]. The values of reliability index of a redundant group of n units, $R(n)$, are calculated as:

$$R_k(n) = 1 - (1 - r_k)^{n+1} \tag{1}$$

Corresponding result are presented in the following Table I.

For each unit, let us write a two-dimensional generation function of the form

$$\begin{aligned} \psi(y, x_k) &= R_k(0) + R_k(1) x_k^1 y^{C_k} + R_k(2) x_k^2 y^{2C_k} \\ &+ R_k(3) x_k^3 y^{3C_k} + \dots\dots\dots\dots\dots\dots + .R_k(n) x_k^n y^{nC_k} \end{aligned} \tag{2}$$

where $R_k(n)$ denotes reliability index of unit k with x spares; the power of x_k means how many spare units of type k are used; the power of y shows how much is spent for spares.

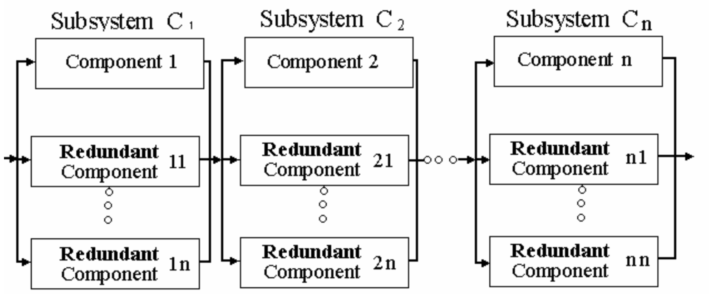

Figure 2: Multiple parallel-series units system with redundancy

Table I: Different reliability values for power units system

Redundant Elements n	$R_1(n)$	$R_2(n)$	$R_3(n)$
1	0.990	0.960	0.985
2	0.999	0.992	0.998
3	0.999	0.998	0.999
4	0.999	0.999	0.999
.	.	.	.
....	...	...	...

Substituting the input data from above, we get

$$\psi(y, x_1) = 0.99 + 0.999x_1^1 y^1 + 0.9999x_1^2 y^2$$
$$+ 0.9999x_3^3 y^3 + ...$$

$$\psi_2(y, x_2) = 0.96 + 0.992x_2^1 y^3 + 0.998x_2^2 y^6$$
$$+ 0.9999x_2^3 y^9 + ...$$

$$\psi_2(y, x_3) = 0.98 + 0.998x_3^1 y^5 + 0.9997x_3^2 y^8$$
$$+ 0.99997x_3^3 y^{11} + ...$$

First stage . We characterize the system by the product of these two polynomials with ordering by increasing power of y, secondly by order of x_1, and then by x_2.

$$\psi^*(y, x_1) = 0.99 * 0.96 + 0.999 * 0.992x_1^1 y^1 + 0.9999x_1^2 y^2$$
$$+ 0.9999x_3^3 y^3 + ...$$

$$\psi_1(y, x_1) = 0.99 + 0.999x_1^1 y^1 + 0.9999x_1^2 y^2$$
$$+ 0.9999x_3^3 y^3 + ...$$

$$\psi_2(y, x_2) = 0.96 + 0.992x_2^1 y^3 + 0.998x_2^2 y^6$$
$$+ 0.9999x_2^3 y^9 + ...$$

$$\psi_3(y, x_3) = 0.98 + 0.998x_3^1 y^5 + 0.9997x_3^2 y^8$$
$$+ 0.99997x_3^3 y^{11} + ...$$

Second stage. We characterize the system by the product of these two polynomials with ordering by increasing power of y, secondly by order of x_1, and then by x_2.

$$\psi^*(y, x_1, x_2) = \psi(y, x_1) * \psi(y, x_2)$$
$$= 0.9 * 0.8 + 0.8 * 0.99x_1^1 y^1 + 0.999 * 0.8x_1^2 y^2$$
$$+ 0.9 * 0.96x_2^1 y^3 + 0.9999 * 0.8x_1^3 y^3 + 0.99 * 0.96x_1^1 x_2^1 y^4 ..$$
$$+ 0.99999 * 0.8x_1^4 y^4$$

Let us notice that the new polynomial is now with three-dimensional only for two components because it is tracking both x_1 and x_2. How can one `read' the polynomial $\psi(y, x_1, x_2)$?

The coefficients of polynomial represent the reliability of redundancy variants for the considered system. The power of x_1 and x_2 means the number of redundant units of the first (second) type, and the power of y means the total spare units cost (in some units). Notice that the absence of x_1 and x_2 in some terms means that the power equals zero, i.e. there is no redundant unit of this type. We remember that we need to find the so-called nominated sequence, i.e., a sequence of the spare unit allocation that is characterized by monotone increase of system reliability index with growth of the total system cost.

Third stage. We characterize the global system by the product of these two polynomials ψ(y, x_1, x_2) and ψ(y, x_3) , with ordering by increasing power of y, secondly by order of x_1, x_2 and then by x_3. Let us notice that the new polynomial is four-dimensional only for three components because it is tracking both x_1 , x_2 and x_3. How can one `read' the polynomial ψ(y, x_1, x_2) and ψ(y, x_3)? As cited above, the general polynomial function is:

$$\psi_{System}(y,x_1,x_2,x_3) = \psi^*(y,x_1,x_2) * \psi(y,x_3)$$

$$= \left[\underbrace{\begin{array}{l} 0.9*0.8 + 0.8*0.99x_1^1 y^1 + 0.999*0.8x_1^2 y^2 \\ +0.9*0.96x_2^1 y^3 + 0.9999*0.8x_1^3 y^3 + 0.99*0.96x_1^1 x_2^1 y^4 .. \\ +0.99999*0.8x_1^4 y^4 \end{array}}_{\psi^*(y,x_1,x_2)} \right] *$$

$$\left[\underbrace{\begin{array}{l} 0.98 + 0.998x_3^1 y^5 + 0.9997x_3^2 y^8 \\ +0.99997x_3^3 y^{11} + ... \end{array}}_{\psi_3(y,x_3)} \right]$$

The general procedure of GGS for *N* component are:

$$\psi_{System}(y,x_1,x_2,x_3) = \underbrace{\psi^*(y,x_1,x_2) * \psi(y,x_3)}_{\psi^*(y,x_1,x_2,x_3)} * \psi(y,x_4) *$$

$$... \underbrace{\psi^*(y,x_5,x_6) * \psi(y,x_{n-1})}_{\psi^*(y,x_1,x_2,...,x_{n-2})} * ... \underbrace{\psi^*(y,x_{n-3},x_{n-4}) * \psi(y,x_n)}_{\psi^*(y,x_1,x_2,...,x_{n-1})}$$

$$\overline{\Omega}\Sigma(y,x_1,x_2,x_n) = \prod_{i=1}^{n-1} \underbrace{\psi(y,x_1,x_2,...,x_{n-1}) * \psi(y,x_n)}_{\psi(y,x_1,x_2,...,x_n)} \quad (3)$$

The usage of generating function allows us to construct a sequence of solutions (as a kind of Pareto set) exhibiting some attributes of the optimal solution (the numbers of spares which corresponds to each particular pair or impair (reliability- cost).

3 AVAILABILITY ESTIMATION BASED ON UGF MODIFIED TO GGS METHOD FOR PRIOP

This procedure, with the use of a standard generating function UGF is appropriate for the case presented above, as soon as `transformations' over various parameters are such as products (for probabilities) and summations (for costs) as in [9]. However, similar enumerating problems are not restricted by these operations over parameters. Let us now suggest a more `computerized' presentation of the procedure keeping further generalization of the method in mind. To make the presentation free from any customary terminology, to avoid any confusion and to make it more mnemonic, let us turn to the terminology used in ancient Algerian power industry as (power transformer industry). The largest mechanics and electric units was called a legion. Each legion comprised cohorts. A cohort comprised several different maniples - `Coiling, Magnetic circuit ect..' Industry sections units that differ from all others. Cohorts might be identical, i.e. consisting of the same type and the same number of maniples. (*In the current context cohorts are identical*.). Legions might differ by the number of cohorts. (This presentation is similar to the all evolutionary program ring language (object oriented).

By using this terminology, the parameters in our optimization problem are: (reliability indices. cost, the number of redundant units, types or version of the units etc.) can be called maniples. So, in our case, we have three types of maniples:

1- ***Number of redundant units (x).***
2- ***Reliability index (R).***
3- ***Cost of unit (c).***

A cohort, **C**, consists of the maniples mentioned above for some particular ***x*** and corresponding ***R*** and ***c***. So the *jth* cohort is:

$$C_j = \left\langle R_j, c_j, x_j \right\rangle \tag{4}$$

A legion is formed by a set of cohorts. Since we will use several different legions, it is reasonable to supply each cohort with its '*identification legion*'. For instance, if cohort *j* belongs to legion *k*, we will denote this as:

$$C_{kj} = \left\langle R_{kj}, c_{kj}, x_{kj} \right\rangle \tag{5}$$

However, thus legion k consists of the following cohorts as:

$$\begin{aligned} L_k &= C_{k1}, C_{k2}, \ldots, C_{kN} \\ &= \left\{ \left\langle R_{k1}, c_{k1}, x_{k1} \right\rangle, \left\langle R_{k2}, c_{k2}, x_{k2} \right\rangle, \ldots, \left\langle R_{kN}, c_{kN}, x_{kN} \right\rangle \right\} \end{aligned} \tag{6}$$

Let us consider now two different legions, the first legion is denoted by L_1 of N_1 cohorts and the second by L_2 of N_2 cohorts. Let us determine that `***Interaction***' of two different legions leads interaction of all possible pairs of cohorts $C_{1j}$ with $j=1, 2, \ldots, N_1$, and $C_{1s}$ with $s=1, 2, \ldots, N_2$. The procedure of the cohort *interaction* will be denoted by ` **δ** '. In other words, we will have $N_1 * N_2$ pairs of interacting cohorts, expressed by:

$$\begin{cases} C_{11}\delta C_{21} & C_{12}\delta C_{21} & \ldots & C_{1N_1}\delta C_{21} \\ C_{11}\delta C_{22} & C_{12}\delta C_{22} & \ldots & C_{1N_1}\delta C_{22} \\ \ldots\ldots\ldots & & \ldots & \ldots\ldots\ldots \\ \ldots\ldots\ldots & & \ldots & \ldots\ldots\ldots \\ C_{11}\delta C_{2N_2} & C_{12}\delta C_{2N_2} & \ldots & C_{1N_1}\delta C_{2N_2} \end{cases} \tag{7}$$

Interaction of a pair of cohorts leads to maniple `interaction'. We remark that each maniple of the first cohort interacts with the same type of maniple of the second cohort, i.e. *R* with *R, c* with c, and x with x. Maniples of each type interacts in their own way. For our case, the interaction of maniples of type R is denoted by :

- $\boldsymbol{\delta_R}$

maniples of type c by:

- $\boldsymbol{\delta_c}$

and maniples of type x by:

- $\boldsymbol{\delta_x}$.

The interactions for these particular types of maniples conduct to the following expressions:

$$R_{Maniple,New=}R_{1j}\delta_R\ R_{2k} = R_{1j} * R_{2k} \tag{8}$$

$$c_{Maniple,New=}c_{1j}\delta_c\ R_{2k} = c_{1j} + c_{2k} \tag{9}$$

$$X_{Maniple,New=}x_{1j}\delta_x\ R_{2k} = \langle x_{1j}, \mathrm{x}_{2k} \rangle \tag{10}$$

In this particular case, as the result of two cohort interaction, we obtain a new cohort with new maniples. Each maniple presents the same resulting parameters as in the polynomial $\psi_\Sigma(y, x_1, x_2)$ shown above. However, if maniple interaction differs from these three types mentioned above, pure polynomial presentation with consequent multiplications fails. A simple example will be considered later.

Interaction of two legions produces cohorts which can be undominated or dominated in the sense mentioned before. A domination of cohort C_1 over cohort C_2 will be denoted by $C_1 \|\succ\| C_2$. The procedure of building a sequence of undominated cohorts can be described as the following algorithm:

4 THE GENERALIZED GENERATING SEQUENCE GGS ALGORITHM

STEP.1

Initialisation

STEP.2

For $j=1$ to N

After iteration of cohort, ***say***

C_{11} and C_{21} we obtain the First resulting cohort , ***say***

C_1^* Save the first cohort

STEP.3

Repeat

Then after iteration of cohort, ***say***

C_{12} and C_{21} we obtain the second resulting cohort , ***say***

C_2^* Save the second cohort

Testing (Between Cohorts) If

cohorts , ***say***

$C_1^* \| \succ \| C_2^*$

Then

C_2^*, is sent to trash

cohorts , ***say***

$C_2^* \| \succ \| C_1^*$

C_2^*, is saved and

C_1^*, is sent to trash

Then If neither

$C_2^* \| \succ \| C_1^*$ *,nor* $, C_1^* \| \succ \| C_2^*$ both of them are:

cohorts undominated

C_2^*, C_1^*, are saved

STEP.3

Repeat The procedure (For **STEP.1** TO **STEP.2**)

STEP.4

Procedure Filtration

The new cohort appear and excluding several cohorts (previously saved).

STEP.5

cohorts , ***say***

(a) *dominated* new cohort.

(b) *undominated* new cohort .

(c) *undominated* and *dominates* new cohort are over two previous cohorts.

cohorts , ***say***

Save, the final legion obtained as the result of interaction of all legions (after ***N-1*** pair interaction of *legions*) after the final `filtering' contains the entire *undominated* sequence (at least in the predetermined range).

End

We described interaction of two legions above. It is possible to arrange a simultaneous interaction of several legions. The principal idea remains the same. In this case we have three expressions:

$$R_{Maniple,New=}\left[\left\langle R_{1k_1}\delta_R\ R_{2k_2}\right\rangle\delta_R\ R_{3k_3}....\delta_R\ R_{Nk\,N}\right]$$
$$=\prod_{1\leq s\leq N} R_{sk_s} \tag{11}$$

$$c_{Maniple,New=}\left[\left\langle c_{1k_1}\delta_c\ c_{2k_2}\right\rangle\delta_R\ c_{3k_3}....\delta_c\ c_{Nk\,N}\right]$$
$$=\sum_{1\leq s\leq N} c_{sk_s} \tag{12}$$

$$x_{Maniple,New=}\left[\left\langle x_{1k_1}\delta_x\ x_{2k_2}\right\rangle\delta_x\ x_{3k_3}....\delta_x\ x_{Nk\,N}\right]$$
$$=\underset{1\leq s\leq N}{\delta_x}\ x_{sk_s} \tag{13}$$

The problem considered is an bi-objective optimization which in is to construct undominated sequence of the system configurations (trade-off ` ***Reliability-Cost*** '). This allows engineering designer to choose the best solution for required reliability or admissible system cost.

In this case interaction of maniples $\dot{\mathbf{g}}$ is defined as:

$$\dot{g}_{Maniple,New=}\ \dot{g}_1\,\delta_{\dot{g}}\ \dot{g}_2 = \frac{1}{\left\langle 1/\dot{g}_1\right\rangle + \left\langle 1/\dot{g}_2\right\rangle}$$
$$=\left(\frac{1}{\dot{g}_1}+\frac{1}{\dot{g}_2}\right)^{-1} \tag{14}$$

For any mixed configuration as series-parallel one. Notice that interaction of legions for parallel and series structures are different. Let us introduce new notation: π is an interaction symbol for legions interacting in series structure, and σ is an interaction symbol for parallel structure. Form reliability index interactions of maniples can be written by the following expressions:

$$R_1\pi R_2 = R_1 * R_2 \tag{15}$$

and

$$R_1\sigma_R R_2 = 1-\left\langle 1-R_1\right\rangle * \left\langle 1-R_2\right\rangle \tag{16}$$

Concerning cost in this case, is given by the following expression:

$$c_1\sigma_c\, c_2 = c_1\pi_c\, c_2 = \left\langle c_1 + c_2\right\rangle \tag{17}$$

So, the entire system final legion can be written in the following form:

$$L_{\Sigma} = L\sigma\langle L_2 \pi \ L_3 \rangle \quad (18)$$

5 ILLUSTRATIVE EXAMPLE

Now let us consider several simple examples where GGS is really effective for enumerating process and standard generating function UGF does not work at all [10]. Let consider a system containing *n* subsystems C_i ($i = 1, 2, \ldots, N$) in series arrangement as represented in figure 3.

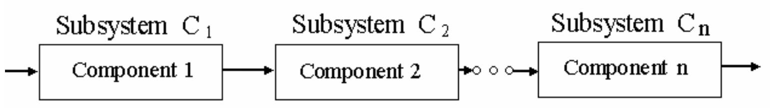

Figure 3: SERIES STRUCTURE

Each unit has exponential distribution of time to failure. Each unit is characterized by mean time between failures (***MTBF***), the maniple **ġ**, and cost, ***c***. There are several vendors who might be potential suppliers, each of which can deliver unit of each type with different values of parameters **ġ** and c given in the following Table II from Figure.4.

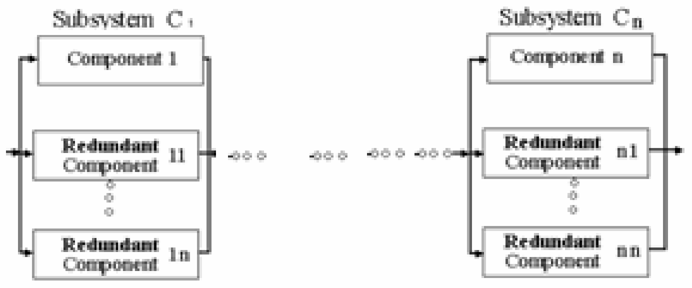

Figure 4: REDUNDANT STRUCTURE

Table II: Different values of paramaters

$\dot{g}_1$	C_1	$\dot{g}_2$	C_2	$\dot{g}_3$	C_3
1.00	1.00	1.00	1.50	1.50	1.00
2.00	3.00	3.00	2.50	3.00	1.50
3.00	4.00	0.00	0.00	0.00	0.00

R_1	C_1	R_2	C_2	R_3	C_3
0.90	1.0	0.70	1.00	0.55	2.00
0.95	1.5	0.72	1.50	0.70	2.50
0.98	2.0	0.00	0.0	0.00	0.00

It is clear that each **ġ**$_1$, C$_1$, **ġ**$_2$, C$_2$, and **ġ**$_3$, C$_3$: represents the corresponding legions. As shown the interaction of maniples **ġ** is :

$$\dot{g}_{Maniple,New=}\ \dot{g}_1\,\delta_{\dot{g}}\,\dot{g}_2 = \left(\frac{1}{\dot{g}_1}+\frac{1}{\dot{g}_2}\right)^{-1}$$

and interaction of maniples remains the same. Interaction of *legions 1* and *2* gives:

$$\begin{cases} \dot{g}_2^* = \dot{g}_{11}\delta_{\dot{g}}\dot{g}_{21} = (1+1)^{-1} = 0.5 \\ \dot{g}_2^* = \dot{g}_{11}\delta_{\dot{g}}\dot{g}_{22} = \left(1+\frac{1}{3}\right)^{-1} = 0.75 \\ \ldots\ldots \qquad \ldots\ldots \\ \ldots\ldots \qquad \ldots\ldots \\ \dot{g}_6^* = \dot{g}_{13}\delta_{\dot{g}}\dot{g}_{22} = \left(\frac{1}{3}+\frac{1}{3}\right)^{-1} = 1.5 \end{cases}$$

$$\begin{cases} c_1^* = c_{11}\delta_c c_{21} = 1+1.5 = 2.5 \\ c_2^* = c_{11}\delta_c c_{22} = 1+2.5 = 3.5 \\ \ldots\ldots \qquad \ldots\ldots \\ \ldots\ldots \qquad \ldots\ldots \\ c_6^* = c_{13}\delta_c c_{22} = 4+2.5 = 6.5 \end{cases}$$

We omitted intermediate ***steps*** in the procedure, preferring to give the final results in the following table Table III.

Table III. Results of interaction between legions 1&2 new legions

Maniple	Interacted Cohorts [New Cohorts]					
	C_{11}, C_{21}	C_{11}, C_{22}	C_{12}, C_{21}	C_{12}, C_{22}	C_{13}, C_{121}	C_{13}, C_{22}
	New c_1^*	New c_2^*	New c_3^*	New c_4^*	New c_5^*	New c_6^*
ġ	0.50	0.75	0.66	1.20	0.75	1.50
c	2.50	3.50	4.50	5.50	5.50	6.50

From the result (Table III), one can see that the new maniple $\underline{\underline{c_2^*}}$ dominates over c_3^*, c_4^* and c_5^*. So, the resulting legion actually appear and contains only $\underline{\underline{c_1^*, c_2^*, c_4^*}}$ and $\underline{\underline{c_6^*}}$.

The final legion after filtering is found in a similar way as shown in Table IV.

Table IV: Results after filtering with dominated cohorts

Maniple	Interacted Cohorts [New Cohorts]					
	C_2^* , C_{31}	C_2^* , C_{32}	C_4^* , C_{31}	C_4^* , C_{32}	C_6^* , C_{31}	C_6^* , C_{32}
	New C_3^{**}	New C_4^{**}	New C_5^{**}	New C_6^{**}	New C_7^{**}	New C_8^{**}
ġ	0.50	0.60	0.66	0.87	0.75	1.00
c	4.50	5.00	6.50	7.00	7.50	8.00

The undominated cohorts are $C_2^{**}, C_3^{**}, C_4^{**} C_5^{**}, C_6^{**}$ and C_8^{**} . Only cohort C_7^{**} is dominated. Table IV allows one to choose the best variant of the system configuration. Now let us perform simple computations. Results of type π interaction of legions 2 and 3 are presented in Table VI. From this table one can see that cohort C_3^* is dominated by cohort C_2^*.

Table V Results of type π between legions

Maniple	Interacted Cohorts [New Cohorts]			
	C_{21}, C_{31}	C_{21}, C_{32}	C_{22}, C_{31}	C_{22}, C_{32}
	New C_1^*	New C_2^*	New C_3^*	New C_4^*
R	0.895	0.910	0.902	0.916
c	3.000	3.500	3.500	4.000

Table VI Results of type σ between legions

Maniple	Interacted Cohorts [New Cohorts]			
	C_1^* , C_{11}	C_1^* , C_{12}	C_1^* , C_{13}	C_2^* , C_{11}
	New C_1^{**}	New C_2^{**}	New C_3^{**}	New C_4^{**}
R	0.806	0.850	0.877	0.819
c	4.000	4.500	5.000	4.500
Maniple	C_2^* , C_{12}	C_2^* , C_{13}	C_4^* , C_{12}	C_4^* , C_{13}
	New C_5^{**}	New C_6^{**}	New C_7^{**}	New C_8^{**}
R	0.865	0.892	0.824	0.970
c	5.000	5.500	5.000	5.000

Maniple	Interacted Cohorts [New Cohorts]					
	C_2^* , C_{31}	C_2^* , C_{32}	C_4^* , C_{31}	C_4^* , C_{32}	C_6^* , C_{31}	C_6^* , C_{32}
	New C_3^{**}	New C_4^{**}	New C_5^{**}	New C_6^{**}	New C_7^{**}	New C_8^{**}
ġ	0.50	0.60	0.66	0.87	0.75	1.00
c	4.50	5.00	6.50	7.00	7.50	8.00

Table VI represent the result of interaction of type σ and the new legion appear after exclusion of cohort with legion one. It’s easy to find the final legion after following the filtration given by the program in Java (see algorithm) the final legions include five legion with minimal cost and high availabilities.

6 CONCLUSION

In this work the generalized generating sequence (GGS) method was applied for solving some practical enumerating industry problems dealing with conditional bi-objective optimization.

A new enumerating algorithm is suggested. This algorithm is convenient for a large industry programs implementation.

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