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DESIGN OF THE POSITION CONTROLLER USING THE GENERALIZED METHOD OF THE REQUIRED DYNAMICS

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Abstract: In this paper we present a simple method for design of the controller with two degrees of freedom using the generalized method of the required transfer function. Using this method, only one controller is designed.

Keywords: PID controller, 2DOF, IMC structures, robustness, stable and unstable systems

1 INTRODUCTION

Older methods for the synthesis of the PID controller for SISO systems [1] supposed the design of the controller with respect to the control signal or to the disturbances. Modern methods of SISO control suppose an appropriate design for both control and disturbances. Nowadays there are available some methods [2], [3], [4],... for design of the controller with satisfactory properties both for control and for disturbances. Most of these methodologies works like follows: Appropriate controller for control and for disturbances are designed independently. Appropriate disturbance controller usually comprises relatively large overshoot for the step change of the control signal. This overshoot is suppressed using such a filter that control system behaviour is the same – similar like the control signal with controller control. In the praxis are preferred those methods [5], where the structure of the PID controller is defined by the ISA standard

$$G_c(s): \quad U(s) = K_R\left((bW(s) - Y(s)) + \frac{1}{T_I s}E(s) + (cW(s) - Y(s))\frac{sT_D}{1 + sT_f}\right) \tag{1}$$

where b and c are weight coefficients of the proportional and derivative parts of the control signal. Most of the methods for the controller synthesis is based on the optimization of the functional $J = \int_0^\infty f(e(t))dt$, where $e(t)$ is the regulation error due to the step change of the control signal and/or disturbance. This paper describes operation of the systems with the two degrees of freedom and inference of the relationship which simplifies the controller synthesis. The result of the analysis of some randomly chosed papers concentrated on the design of the controller appropriate for both control and disturbances is following: When the controller has appropriate properties for both control and for disturbances, then the essence of the particular method can be reduced to the design using the inverse dynamics method („reduce everything is possible"). The only difference is that classical inverse dynamics method „reduce" transfer function of the original controlled system. When something is impossible to reduce - is there any instability, then stabilize it first and then reduce. This is an essence of the „properly" designed IMC structures. Our presented method supposes „reduction" of the modified transfer function of the controlled system. Let us note that we don't change the structure and parameters of the controlled system, but its behaviour. Result of the analysis is the understanding how to assign the disturbance controller (4) to the (1) calculated using the generalized method of the wished dynamics to comply both requirements for control and for disturbance suppression.

2 INFERENCE OF THE FORMULA

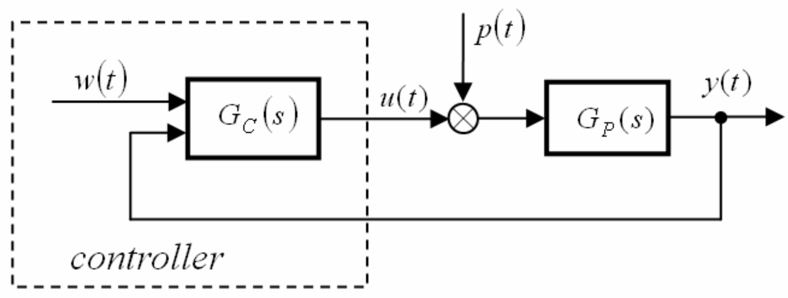

Fig.1 SISO control system

Let us suppose the simple SISO system according to the Fig. 1., where $G_P(s)$, is the transfer function of the stable controlled system without the transport delay (for the purpose of an explanation of the 2DOF principle). Most of the methods for controller synthesis suppose that we are able to assign the model $M(s) \hat{=} G_P(s)$ to the controlled system. Let us suppose that the controller is designed using the method of the inverse dynamics

$$G_C(s) = \frac{1}{\lambda s M(s)} = \frac{1}{\lambda s G_P(s)} \tag{2}$$

The time constant λ represents the required dynamics of the control (this term is sometimes mentioned as *dominant time constant, throughput band, cut-off frequency* as necessary). We suppose that this controller is supplied with the filter of realization that in the contrary of the IMC structures can not influence the throughput spectrum. On the contrary, when considering the interaction of the second kind in the controller (1), we will design $T_f = f(\lambda)$ (that means T_f and T_D, T_I can be comparable). In the formula (2) and in following we omit this filter. We clarify the principles here. For the Fig. 1. and the controller (2) we can write

$$Y(s) = \frac{1}{\lambda s + 1} W(s) + \frac{\lambda s}{\lambda s + 1} G_P(s) P(s) \tag{3}$$

This result, freely speaking, means: The transfer function of the control is not a function of the controlled system transfer function. Then the $G_P(s)$ can be also the second order underdamped system. Theoretically we are able to achieve the any arbitrary short response without the overshoot. From the second part of the (3) it is clear that for the disturbance transfer function it is more complicated. Dynamics of the disturbance depends on the transfer function of the controlled system. When we want to achieve the disturbance transfer without overshoots, the controlled system has to be stable and cannot contain the second order underdamped systems. It is clear that we cannot change the $G_P(s)$, but we can change its „behaviour". In the introduction we mentioned two groups for controller design – for control and for disturbance signals. It can be stated that those methodologies: a) only little resemble the design using the inverse dynamics method and b) improvement of the disturbance dynamics was achieved mainly by changing the value of λ. In the particular case it is a controller gain. Control transfer function of such controller can be improved with an appropriate filter in the control part (see Fig. 2).

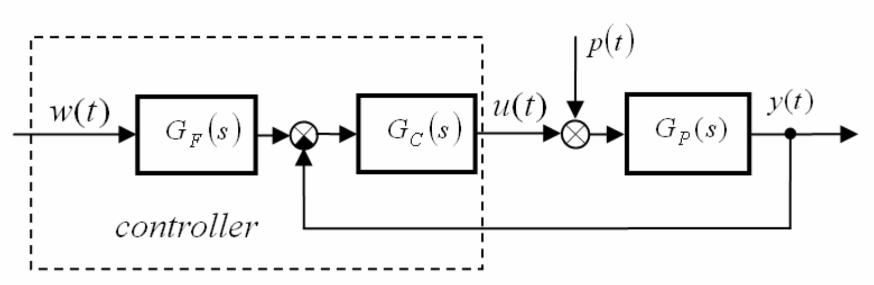

Fig.2 SISO control system with a filter in the control part

Another, modern method to improve both dynamics of the control and disturbances is to „modify“ the controlled system transfer function, i.e. the behaviour of the $G_P(s)$. We want to achieve that the controlled system behaviour is the same as the system with appropriate properties. We require that the $M(s)$ is stable, its transient response is without overshoots and fades in predetermined time. When we achieve, that $G_P(s)$ (see Fig. 1) will behave like the $M(s)$, we can use the relationship $G_C(s)=\frac{1}{\tilde{\lambda} s M(s)}$ for inverse dynamics. The relationship (3) will change to the $Y(s)=\frac{1}{\tilde{\lambda} s+1}W(s)+\frac{\tilde{\lambda} s}{\tilde{\lambda} s+1}M(s)P(s)$. Method of the inverse dynamics did change to the method of the required model. For the method itself it is just cosmetic change. More appropriate term is the method of the „required control transfer function“ in the form $G_{WY}(s)=e^{-sD}\left(\lambda s+e^{-sD}\right)^{-1}$. When we require the control transfer function in the form $G_{WYS0}(s)=e^{-sD}(\lambda s+1)^{-1}$ it will result in Smith predictor. When we require the control transfer function in the form $G_{WY0}(s)=(\lambda s+1)^{-1}$, the controller will correspond to the classical method of the inverse dynamics. This problem will be described in an another paper. For now, let us note that when we want to implement in the relationship (2) both the required model of the controlled system and the required control transfer function, we have to infer the control for the disturbance transfer function. Otherwise the $G_P(s)$, respectively $M(s)$ will be „reduced“ and disappears from the formula.

The relationship (3) together with modifications of the method of the inverse dynamics is the basis for almost all modern methods for controller design: Smith predictor, IMC (stable, unstable) structures, 2DOF, ...

For example: the equation $R(s)(M(s)=G_P(s))=G_{WY0}(s)(1+R(s)(M(s)=G_P(s)))$ is the basis for classical IMC structures. But also the equation written in this form enables to design „better = faster“ control system than classic IMC structures. IMC structures contain usually instead of the $G_{WY0}(s)$ the filter $F(s)=(s\tau+1)^{-n}$.

Let us suppose that both the model $M(s)=M_0(s)e^{-sD}$ and the controlled system $G_P(s)=S_0(s)e^{-sD}$ contain the same time lag D. $M_0(s)$ is stable and $S_0(s)$ is in general unstable. We consider the both systems of the same order. Using the algebra of the systems and knowledge from the basics of the control theory, we can in addition consider also the cases where the order of $M_0(s)$ is less than order of the $S_0(s)$. When required disturbance transfer function is in the form $G_{PY}(s)=\frac{\lambda s}{\lambda s+e^{-sD}}\frac{M(s)}{\beta}$ then solving the $\frac{S(s)}{1+G_C(s)S(s)}=G_{PY}(s)$ we obtain the controller for disturbances in the form of

$$G_c(s) = \frac{\beta}{\lambda s M_0(s)} + \left(\frac{\beta}{M(s)} - \frac{1}{G_P(s)} \right) = G_{ID}(s) + V(s) \tag{4}$$

Notes to the relationship (4):

1. The controller in its entirety used to be stable and realizable and to all intents it can be recalculated to the form (1). It is obvious that the filter used to be added to the derivative part. Parameter β provides the stability of the inner feedback. Its operation can be formulated in the form: Every time the law of preservation of something applies. E. g. When we want to decrease the time constant to its half, we can do it, but also the gain of the system will be decreased.
2. When the resulted controller used to have the form (1), for the practical purposes it is necessary to suitably choose the order of the $M(s)$ - maximum two. That means that when the $G_P(s)$ is of the higher order, it is necessary to appropriately damp it using the gain of the open system. This is probably the essential difference from the IMC structures. That means: required resulting transfer function will be achieved using the combination of the damping of the open circuit gain and modifying the properties of the controlled system.
3. When we suppose that the time lags of the model and controlled system are equal, stability of the system is given by the solution of the *small characteristics equation* $m.ch.r.(s) = \lambda s + e^{-sD} = 0$. Required dynamics is not freely chosed but calculated so that quality of the system corresponds to the „1, or 5% exponential" respectively (similar to the transient response of the second order system with the 1% or 5% overshoots). Digital control systems are the special case of the systems with the time lag. This reason enables to use the small characteristics equation also for design of the sampling rate (input sampling rate, sampling rate of the PS controller and for calculating digital controller to the corresponding continuous one).
4. When we consider (besides others), that transport delay of the system is changing within the interval $D \in \left\langle D_M, D_{M\,MAX} \right\rangle$ and the transport delay of the model is D_M, the stability of the system is determined by the solution of the *big characteristics equation* $v.ch.r.(s) = 1 + s\lambda - \left(e^{-sD_M} - e^{-sD}\right) = 0$. Solution of this equation belongs to the areas of „robustness" or the Smith predictor($D \neq D_M$), respectively.
5. In an accordance with the IMC structures also here the factorization has to be used: $G(s) = G^+(s)G^-(s)$. Everywhere where the result is unrealisable there used to be instead of $G_P(s)$ or $M(s)$ used the $G_P^+(s)$, or $M^+(s)$ respectively.
6. The controller designed according the (4) provides also good properties for control and for disturbances. Such the controller is in the literature denoted as the controller with two degrees of freedom and usually it follows certain dynamics for the control and certain one for the disturbances. According (4) it is more appropriate to state that we have two requirements: first we require that control system behaviour is the same as in the model and then we require certain behaviour of the whole control system. Such understanding of the term 2DOF is used here.

3 EXAMPLES

Example 1. Following example comes from [2] and its transfer function was reduced to the first order system: $G_P(s) = (1+10s)^{-1}$. Controller is designed such that error step change at the system input will damps out after, let's say, 5 seconds ($\lambda = 1\,[s]$) and without big overshoot at the same time. The example is completed with a requirement of the minimal size of the action value. This example represents an essence of so called 2DOF in literature. How it works. Controller (1) parameters: a) Method based on dominant pole cancellation [2]: $K_R = 19$; $b = 1$; $T_I = 1.9\,[s]$. (Control step $w(t) = 1(t-5)$ and error $p(t) = 1(t-30)$), time diagrams see Fig.3. b) Generalized method of the wished dynamics: $K_R = 19$; $b = 0.53$; $T_I = 1.9\,[s]$. (Control step $w(t) = 1(t-10)$ and error $p(t) = 1(t-35)$). c) Generalized method of the wished dynamics, with minimized action value: $K_R = 19$; $b = 0$; $T_I = 1.9\,[s]$. Control step response corresponds to the second order system with transfer function $(1+s)^{-2}$; (Control step $w(t) = 1(t-15)$ and error $p(t) = 1(t-40)$). d) Classical inverse dynamics method: $K_R = 10$; $T_I = 10\,[s]$; $b = 1$. (Control step $w(t) = 1(t-20)$ and error $p(t) = 1(t-45)$).

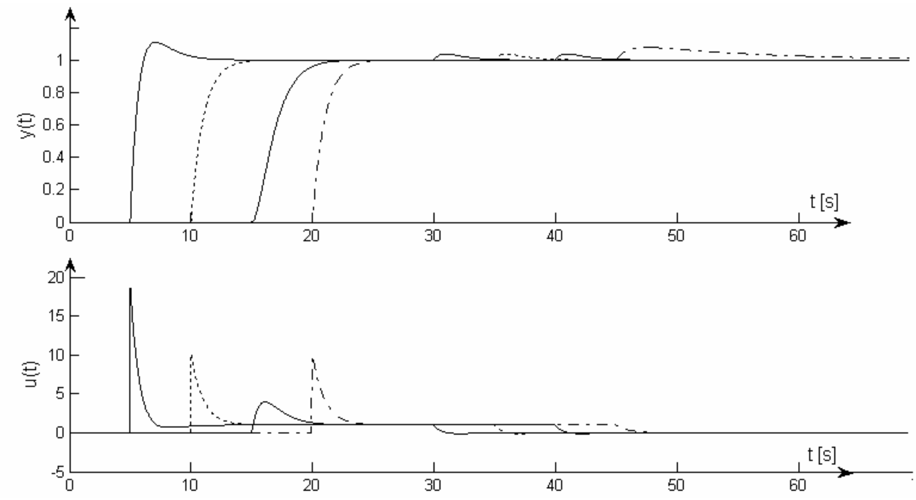

Fig.3 Controlled value and action value courses from Example 1.

Example 2. Second order system $G_P(s) = ((1+s)(1+9s))^{-1}$. This system can be reduced to a previous example, but just for „large" values of the wished dynamics. When decreasing the wished dynamics, we hit the border, usually called *throughput band*. In [3] are two controllers designed for this system: 1) Weighted controller, that is in fact the controller (1) with parameters $b = c = 0$. Remaining parameters of the controller are: $K_R = 18.84$; $T_I = 2.08\,[s]$; $T_D = 0.6\,[s]$. In (1) the filter founds only in derivative part. In [3] is the filter $G_F(s) = (1+0.5s)^{-1}$ superposed before the whole PID controller part. 2) Filtered controller. The difference against the previous one is in $b = c = 1$ and correction element $G_F(s) = (1+1.767s+1.612s^2)/(1+2.512s+2.281s^2)$ in the controller part. From the original example we estimated the wished dynamics to $\lambda = 1.5\,[s]$. Using the method of the wished dynamics we designed following parameters of the controller (1): $K_R = 23$; $b = 0.52$; $T_I = 2.9\,[s]$; $T_D = 0.61\,[s]$; $c = 0$;

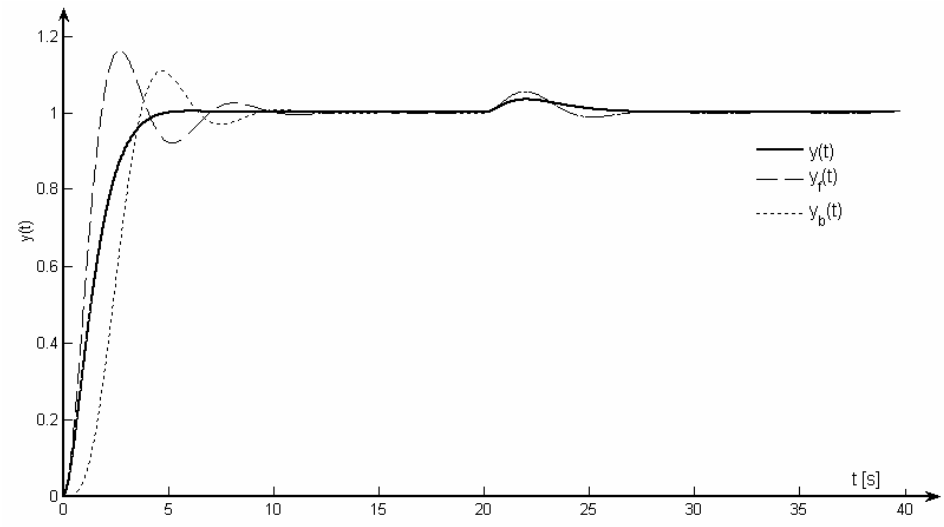

Fig.4 Controlled value for control step $w(t)=1(t)$ and error step $p(t)=1(t-20)$. Diagram labelled with index *f* corresponds to the filtered controller, index *b* to the weighted controller and without the index is the controller designed using the generalized method of wished dynamics.

Example.3. Second order system $G_P(s)=(\frac{1}{3}s^2+\frac{1}{3}s+1)^{-1}$. Controller designed using the classical approach will show damping oscillations on response to the error step change. Control step change response for proper design of the controller will be without overshoots. Using the (4) was controller designed such that response on the control and error step changes will show dynamics corresponding (approximately) to the model: $M(s)=(0.3s+1)^{-1}$ (Order of the system $M(s)$ is one and order of the system $G_P(s)$ is 2. To control the 1^{st} order system the PI controller is sufficient. That means, that D part of the controller will be missing.). This example was chosen so that we can show that our method works also with second order underdamped systems and for unstable systems, e.g. Above-mentioned switched power supply, levitating ball, etc. From the original example [4] we estimated (we want to compare) wished dynamics to value $\lambda=0.3\,[s]$, that corresponds to the half period of the damping oscillations $\approx 0.9\,[s]$. In [4] they designed controller for this system with following parameters: $K_R=8.55$; $b=0.76$; $T_I=0.67\,[s]$; $T_D=0.226\,[s]$; $c=1$. Diagrams were similar to those in [4] just after setting the value of $c=0$. Using the generalized method of wished dynamics we designed the controller (1) with parameters: $K_R=21.2$; $b=0.52$; $T_I=0.58\,[s]$; $T_D=0.19\,[s]$; $c=0$. It is interesting, that in [4] they use the term *throughput band* or *cut-off frequency* instead of "tunable" parameter. But in the example is used the word "let us choose".

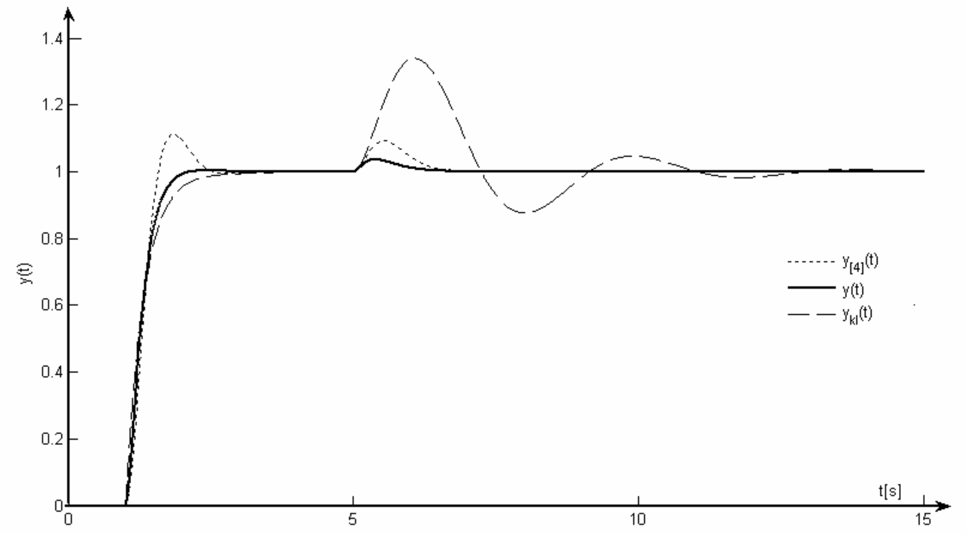

Fig.5 Controlled value diagram for step changes of the control $w(t)=1(t-1)$ and error $p(t)=1(t-5)$ signals. Diagram labelled with index [4] is similar to those in [4], index *kl* corresponds to the controller designed using the inverse dynamics method and without the index corresponds to the controller designed using the generalized method of the wished dynamics.

Example 4. When deriving the equation (2) we assumed that controlled system can be also unstable. Special case of the unstable systems (regarding the equation (4)) is the integrator $1/(Ts)$. It is a good idea to stabilise such a system so that resulting transfer function of the system is e.g.: $b_0/(a_1 s+1)$. In the literature is usually mentioned the integrator together with the transport delay. In [5] is for control of the system $G_P(s)=\dfrac{1}{20s}e^{-5s}$ calculated controller in the form (1) with the following parameters: $K_R=3.14$; $b=0.42$; $c=0.63$; $T_I=18.66\,[s]$; $T_D=1.32\,[s]$. In [5] are the properties of the controller circuit defined by the condition for doubled roots of the characteristic equation, but they didn't mention which.

The form (4) can be used also for control of this system. Here we propose following goal: let the controlled system behaves like the model with the transfer function $M(s)=\dfrac{e^{-5s}}{10s+1}$. Time constant $T=10\,[s]$ is the compromise – the border between the classical controller and the Smith predictor. Second part of the controller is given by the difference $\dfrac{\beta}{M(s)}-\dfrac{1}{G_P(s)}$. This feedback together with the controlled system used to be stable. If we don't want higher orders of derivation than first, it is proper choice the $\beta \overset{!}{=} 2$. Next we proceed like with the IMC structures. Time lag (transport delay) will be replaced with Pade approximation. Unstable parts are not considered. $V(s)=\dfrac{\beta}{M(s)}-\dfrac{1}{G_P(s)}=5s+2$. As this is not realizable system, we have to finalize it with the filter (similar to the IMC structures). Tenth (and less) of the time constant is satisfactory. As we modified behaviour of the controlled system by the realizable feedback, resulted behaviour will not be $M(s)=\dfrac{e^{-5s}}{10s+1}$, but $\hat{M}(s)\hat{=}\dfrac{e^{-5s}}{7.5s+1}$. As $M_0(s)$ we constitute $(7.5s+1)^{-1}$. Wished dynamics λ is determined as follows. Small characteristics equation is : $\lambda s + e^{-5s} = 0$. Response on the control step will contain the overshoot smaller than 1% , when following is true $\frac{\lambda}{\beta} = \frac{T_I}{K_R}$; $\lambda = 2.27 \cdot (D = 5)$. For the $T_I = 7.5\,[s]$ is the gain of the PI controller $K_R = 1.32$. When those parameters will be recalculated to the form (1), it will be: $K_R = 3.3$; $b = 0.4$; $T_I = 18.9\,[s]$; $T_D = 1.5\,[s]$; $c = 0$. By the solving of the small characteristic equation for max. 1% overshoot we also indirectly set the maximum throughput band.

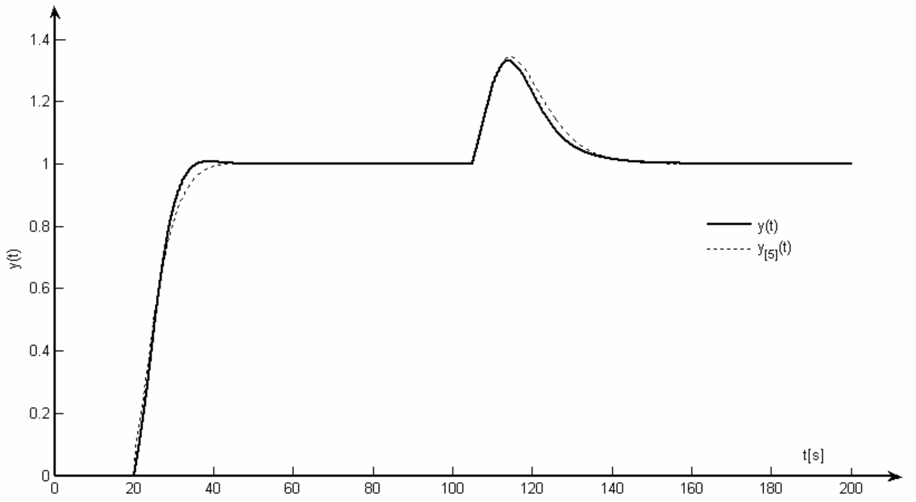

Fig.6 Controlled value diagram for step changes of the control $w(t) = 1(t-15)$ and error $p(t) = 1(t-100)$. Diagram labelled with index [5] corresponds to the controller in [4] and without the index corresponds to the controller designed using the generalized method of the wished dynamics. The difference in results is the consequence of the pole placement. In [5] it is the double pole considered, here we consider a pair of the complex conjugated roots.

4 CONCLUSION

The equation (4) can be used directly for the synthesis of the controller for unstable systems. Its indirect implication is that all the methods for the design of the controllers for unstable systems belongs to the category 2DOF.

An advantage of the (4) is, that when we want to design just the control controller, then we just consider the result as the required controller. Instead of the IMC structures where the λ is so called tunable parameter, the λ in (4) is determined by the requirements on circuit properties.

When we set the parameter $b = 0$ in examples 2 and 3, the response to the control step will be slower. Similarly as in the example 1. the size of the action value will be significantly decreased. Most of the authors didn't mention the size of the action value, nor the realization of the derivative part.

The equation (4) can be viewed also from different point of view. In fact, we have two degrees of freedom, but we have also the two parts (properties) of the controller. First one (in parenthesis) provides the required properties of the system (together with the properties of the error circuit), second part provides required control properties.

Equation (4) was tested also for so called anisochronic systems.

During the controller design based on (4) we can proceed by the two ways:

1. The controller will be designed such that remains „original“ transfer function of the controlled system and let design the controller to it (contents of the parenthesis is zero). When the controlled system is stable, doesn't contain the parts corresponding to the second order underdamped system, we obtain satisfying results also for disturbances. Response to the control step can in practice (and often also in theory) correspond to the response of the first order system. Response on errors is longer in such cases. Dynamics can be changed in such cases with the change of the open circuit gain size.
2. The system dynamics will be altered by the change of the pole placement of the controlled system. The change is achieved by the negative feedback around the controlled system. Change of the pole placement has two reasons:
    a. Stability. Response on the disturbance at the system input is the function of the controlled system. When is the transfer characteristics damped oscillating, also the response will show damped oscillating behaviour. When the transfer function is unstable, it is necessary to stabilise it using the feedback. Response on the disturbance at the system input will then correspond to the response of the stabilised system.
    b. Using the feedback we change also the dynamics of the controlled system. This method we choose when the error step response is too slow.

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