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SIMPLE CONTROLLER TUNING AND NON-OSCILLATORY PLANT IDENTIFICATION

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Abstract

This article is devoted to the simple PI and PID analog and digital controller tuning for non-oscillatory proportional plants of the first and second order with time delay. The approach is a modification of the desired model method and it uses for fine-tune up the only one varying parameter – the controller gain. New identification method is described as well. It enables the acquisition of plant transfer functions in a form suitable for mentioned tuning. All resultant formulas are brought out in easily memorizable forms. The use is shown in the example.

Keywords: PI and PID controllers, time delay, identification

1 INTRODUCTION

Controller tuning methods, which use simple plant models and one tuning parameter, are very popular among engineering professionals. Their advantage is the ability to obtain the desired step response by changing just one tuning parameter (Aström, Hägglund, 1995; O'Dwyer, 2003; Kozáková, Hudzovič, 2001; Vítečková, Víteček, 2006).

This article describes the PI and PID analog and digital controller tuning method, which is based on the desired model method (formerly known as inverse dynamics method) (Vítečková, 1992; Šulc, Vítečková, 2004) and approximation of the non-oscillatory proportional plant with the transfer function in the form

$$G_P(s) = \frac{k_1}{T_1 s + 1} e^{-T_{d1} s} \tag{1}$$

or

$$G_P(s) = \frac{k_1}{(T_2 s + 1)^2} e^{-T_{d2} s} \tag{2}$$

where k_1 is the plant gain, T_j – the time constant, T_{dj} – time delay ($j = 1, 2$).

Table I : Transfer functions of conventional PI and PID controllers

Type	Controller	
	Analog	Digital
PI	$k_P\left(1+\frac{1}{T_I s}\right)$	$k_P\left(1+\frac{T}{T_I}\frac{z}{z-1}\right)$
PID	$k_P\left(1+\frac{1}{T_I s}+T_d s\right)$	$k_P\left(1+\frac{T}{T_I}\frac{z}{z-1}+\frac{T_D}{T}\frac{z-1}{z}\right)$

For identification of non-oscillatory proportional plants of a higher order and with time delay the new simple identification method is proposed (Víteček, Vítečková, 2006). It enables the direct acquisition of the transfer functions in the forms (1) or (2).

2 PI AND PID CONTROLLER TUNING

The controller tuning goes up from the desired model method, i.e. from the desired form of the closed-loop transfer function G_{wy}, which for a control system with a digital controller has the form (Fig. 1) (Šulc, Vítečková, 2004; Vítečková, 1998)

$$G_{wy}(z) = \frac{Y(z)}{W(z)} = \frac{T}{A(z-1) + Tz^{-d}} z^{-d}, \qquad d = \frac{T_d}{T} \tag{3}$$

and for a control system with an analog controller

$$G_{wy}(s) = \frac{Y(s)}{W(s)} = \lim_{T \to 0} G_{wy}(z)\Big|_{z=\mathrm{e}^{sT}} = \frac{1}{As + \mathrm{e}^{-T_d s}} \mathrm{e}^{-T_d s} \tag{4}$$

where A is the varying parameter (it is the inverse open-loop gain), T – the sampling period, T_d – the time delay, d – the discrete relative time delay (for simplicity the integer is supposed).

With regard to neglected small quantization error in this article the concepts discrete and digital are considered as equivalent.

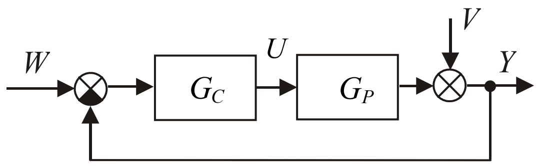

Figure 1: Control system

In Fig. 1 symbols W, U, V and Y are the transforms of desired, manipulated, disturbance and controlled variables, G_C and G_P – the controller and plant transfer functions. The independent variable s is considered for a continuous control system with an analog controller and the independent variable z is considered for a discrete control system with a digital controller.

On the assumption that zeros and undominant poles have negligible effect on the behaviour of a control process the characteristic polynomial of a control system with a digital controller has the form [see (3)]

$$N(z) = A(z-1) + Tz^{-d} \tag{5}$$

The conditions for the approximate marginal non-oscillatory control process can be expressed in the form

$$N(z) = 0, \qquad \frac{\mathrm{d}N(z)}{\mathrm{d}z} = 0 \tag{6}$$

On the basis of the conditions (6) the stable double real pole is obtained

$$z_2 = \frac{d}{d+1} \tag{7}$$

and the corresponding maximum (initial) value of the varying parameter

$$A_d = T(d+1)\left(\frac{d+1}{d}\right)^d \approx (4-\mathrm{e})T + \mathrm{e}T_d \tag{8}$$

For $d \geq 1$ the relative error of the approximation (8) is less than 1,6 %.

From the relation (8) it is possible to obtain the maximum (initial) value of varying parameter A_a for approximate marginal non-oscillatory control process for a control system with an analog controller, i.e.

$$A_a = \lim_{T \to 0} A_d = \mathrm{e}T_d \tag{9}$$

In the above mentioned relations time delay T_d must be considered, which is in the approximate plant transfer function (1) or (2).

The controller transfer function on the basis of the synthesis equation

$$G_C = \frac{1}{G_P}\frac{G_{wy}}{1-G_{wy}} \tag{10}$$

can be obtained, where the desired transfer function (3) is considered for a control system with a digital controller and (4) for a control system with an analog controller.

For a control system with a digital controller the plant transfer function is given by formula (for sampler and holding element)

$$G_P(z) = (1-z^{-1})Z\left\{L^{-1}\left\{\frac{G_P(s)}{s}\right\}\Bigg|_{t=kT}\right\} \tag{11}$$

Depending on the form of the desired control system transfer function (3) or (4) and the plant transfer function (1) or (2) the PI or PID digital or analog controller transfer function (Tab. I) is obtained and at the same time as the formulas for their adjustable parameters. For digital controllers these formulas are simplified. Some of the numerical values for well known constants are substituted (Tab. II) to make memorizing easier.

For a rough estimate of the sampling period T it is possible to use the formulas

$$T = \left(\frac{1}{15} \div \frac{1}{6}\right)\left(4T_1 + T_{d1}\right) \tag{12}$$

or

$$T = \left(\frac{1}{15} \div \frac{1}{6}\right)\left(7T_2 + T_{d2}\right) \tag{13}$$

which come from the known formula (Aström, Hägglund, 1995)

$$T = \left(\frac{1}{15} \div \frac{1}{6}\right)t_{0,95} \tag{14}$$

where $t_{0,95}$ is the time for reaching the value $0{,}95h_P(\infty)$, see Fig. 2.

Tuning procedure:

a) If the non-oscillatory proportional plant transfer function has not the form (1) or (2), then it must be modified to one of these forms.

b) For selected controller type the adjustable parameter values are determined including minimal controller gain for the maximum (initial) value of the varying parameter A. For a digital controller the sampling period must be determined in advance, e.g. on the basis of formulas (12) or (13).

c) The control process will be approximate marginal non-oscillatory. In case if another response is desired, then the control process can be suitable fine-tuned up by changing the controller gain.

Table II: Values of PI and PID controller adjustable parameters

<table>
<tr><td rowspan="2">CONTROLLED PLANT</td><td colspan="4">CONTROLLER
ANALOG $T = 0$
DIGITAL $T > 0$</td><td>INITIAL VALUE OF VARYING PARAMETER A</td><td>NOTE</td></tr>
<tr><td>TYPE</td><td>T_I^*</td><td>T_D^*</td><td>k_P^*</td><td></td><td></td></tr>
<tr><td rowspan="2">$\frac{k_1}{T_1 s + 1} e^{-T_{d1} s}$</td><td>PI</td><td>$T_1 - \frac{T}{2}$</td><td>–</td><td rowspan="4">$\frac{T_I^*}{A k_1}$</td><td>$A \le (4 - e)T + eT_{d1}$</td><td></td></tr>
<tr><td>PID</td><td>$(4 - e)T_1 - T$</td><td>$\frac{T_I^*}{4}$</td><td>$A \le (4 - e)T + eT_{d1} - T_1$</td><td>$T_1 < T_{d1}$</td></tr>
<tr><td rowspan="2">$\frac{k_1}{(T_2 s + 1)^2} e^{-T_{d2} s}$</td><td>PI</td><td>$\frac{\pi}{2} T_2 - \frac{T}{2}$</td><td>–</td><td>$A \le (4 - e)T + eT_{d2} + 1{,}5T_2$</td><td></td></tr>
<tr><td>PID</td><td>$2T_2 - T$</td><td>$\frac{T_I^*}{4}$</td><td>$A \le (4 - e)T + eT_{d2}$</td><td></td></tr>
</table>

3 PLANT IDENTIFICATION

The plant gain k_1 can be determined from the steady state

$$k_1 = \frac{h_P(\infty)}{\Delta u} \tag{15}$$

where Δu is the step of the manipulated variable, $h_P(\infty)$ – the steady-state value of the plant step response.

Two remaining parameters T_1 and T_{d1} or T_2 and T_{d2} of the transfer functions (1) or (2) can be determined on the basis of the equality of times for reaching the value h_x and the complementary areas S for approximate $\widehat{h}_P(t)$ and original $h_P(t)$ plant step responses in accordance with Fig. 2, i.e.

$$\begin{aligned} t_x &= x_j T_j + T_{dj} \\ S &= \int_0^\infty \left[1 - \frac{\widehat{h}_P(t)}{h_P(\infty)} \right] \mathrm{d}t \end{aligned} \tag{16}$$

where t_x is the time of reaching the value h_x for the original plant step response $h_P(t)$, x_j – is the relative time of reaching the value h_x for the approximate plant step response $\widehat{h}_P(t)$ for $T_{dj} = 0$, S – is the complementary area of the original plant step response $h_P(t)$

$$S = \int_0^\infty \left[1 - \frac{h_P(t)}{h_P(\infty)}\right] \mathrm{d}t \tag{17}$$

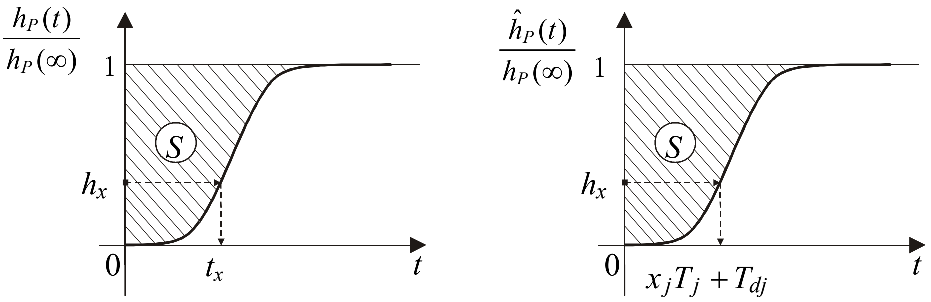

Figure 2: Evaluation of plant step response on the basis of time t_x and complementary area S

The complementary area S for the approximate plant step response $\widehat{h}_P(t)$ can be easily computed for the approximate transfer functions (1) or (2)

$$S = \int_0^\infty \left[1 - \frac{\widehat{h}_P(t)}{h_P(\infty)}\right] \mathrm{d}t = \lim_{s \to 0} \left\{ \frac{1}{s} \left[1 - \frac{G_P(s)}{k_1}\right] \right\} = jT_j + T_{dj} \tag{18}$$

On the basis of the relations (16) and (18) both parameters can be obtained

$$T_j = \frac{S - t_x}{j - x_j}, \quad T_{dj} = \frac{jt_x - x_j S}{j - x_j} \tag{19}$$

The principal problem consists in determining the suitable value h_x. If the value

$$h_x = \frac{\widehat{h}_P(jT_j + T_{dj})}{h_P(\infty)} \tag{20}$$

is chosen, then determination of times T_j and T_{dj} will be ambiguous.

E.g. for $j = 2$ the value h_x must satisfy the inequality

$$0 < h_x < 1 - 3\,\mathrm{e}^{-2} \doteq 0{,}594 \tag{21}$$

The values $h_x = 0{,}28$ and $h_x = 0{,}33$ seem to be suitable. These values are used at the same time for simple experimental identification as well (Vítečková, 1992).

Relative time x_j for $j = 1$ and $h_x = 0{,}28$ and $0{,}33$ can be determined analytically for $T_{dj} = 0$ from the formula

$$x_1 = -\ln(1 - h_x) \tag{22}$$

and for j = 2 iteratively from the formula

$$h_x = 1 - (1 + x_2)\mathrm{e}^{-x_2} \tag{23}$$

The following values were obtained:

$$h_x = 0{,}28; \qquad x_1 = 0{,}3285; \quad x_2 = 1{,}0428 \tag{24}$$

$$h_x = 0{,}33; \qquad x_1 = 0{,}4005; \quad x_2 = 1{,}1796 \tag{25}$$

On the basis of the relations (19), the values (24) and (25) for j = 1 and 2 the formulas in Tab. III were obtained.

The formulas given in Tab. III can be used with the advantage of modifying transfer functions of non-oscillatory proportional plants of a higher order or with time delay in the forms (1) or (2).

Table III : Step responses estimation

$h_x = 0{,}28$	$h_x = 0{,}33$
$T_1 = 1{,}489(S - t_{0,28}) \doteq 1{,}5(S - t_{0,28})$ $T_{d1} = 1{,}489t_{0,28} - 0{,}489S \doteq 0{,}5(3t_{0,28} - S)$	$T_1 = 1{,}668(S - t_{0,33}) \doteq 1{,}67(S - t_{0,33})$ $T_{d1} = 1{,}668t_{0,33} - 0{,}668S \doteq 1{,}67t_{0,33} - 0{,}67S$
$T_2 = 1{,}045(S - t_{0,28}) \doteq 1{,}05(S - t_{0,28})$ $T_{d2} = 2{,}090t_{0,28} - 1{,}090S \doteq 2{,}1t_{0,28} - 1{,}1S$	$T_2 = 1{,}219(S - t_{0,33}) \doteq 1{,}22(S - t_{0,33})$ $T_{d2} = 2{,}438t_{0,33} - 1{,}438S \doteq 2{,}44t_{0,33} - 1{,}44S$

Identification procedure:

a) From the normalized plant step response $h_P(t)/h_P(\infty)$ for the value 0,28 (or 0,33) the time $t_{0,28}$ (or $t_{0,33}$) is determined.

b) Over the normalized plant step response $h_P(t)/h_P(\infty)$ the complementary area S is determined. If the transfer function must be modified in the form (1) or (2), then it is useful the complementary area S to determine on the basis of formula

$$S = \lim_{s \to 0} \left\{ \frac{1}{s} \left[1 - \frac{G_P(s)}{k_1} \right] \right\} \tag{26}$$

c) The time $t_{0,28}$ (or $t_{0,33}$) and the complementary area S substitute in corresponding formulas in Tab. III.

4 EXAMPLE

For the controlled plant with the transfer function

$$G_P(s) = \frac{2(s+1)}{(5s+1)^3}\mathrm{e}^{-4s}$$

it is necessary to determine the PI and PID analog and digital adjustable controller parameters for the transfer function in the form (2) (time constants and time delay are in seconds).

Solution:

The time $t_{0,33}$ = 13,1 s was determined by a digital simulation (see Fig. 3). On the basis of formula (26) the value of complementary area S = 18 s was determined.

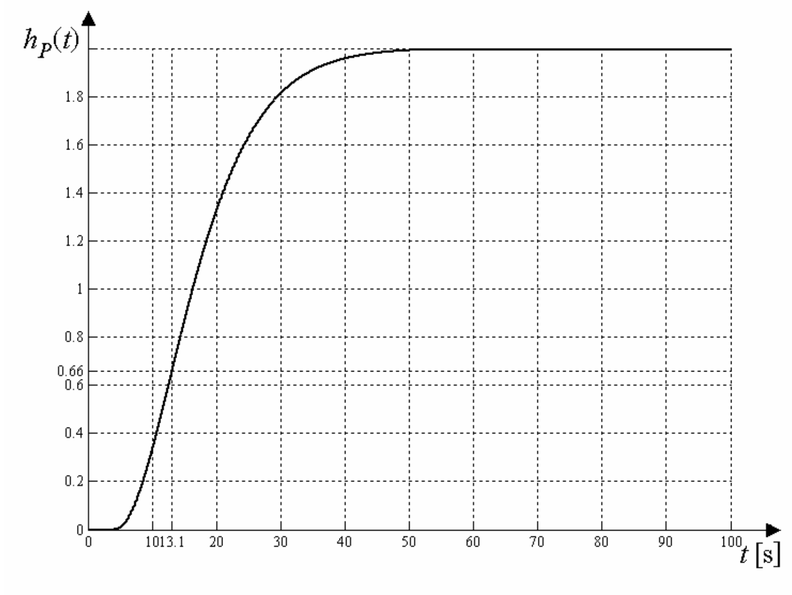

Figure 3: Plant step response – example

For the transfer function (2) on the basis of Tab. III the values of the plant parameters were computed

$$T_2 = 1{,}2(S - t_{0,33}) = 1{,}2(18 - 13{,}1) = 5{,}88\ \mathrm{s}$$
$$T_{d2} = 2{,}4 t_{0,33} - 1{,}4S = 2{,}4 \cdot 13{,}1 - 1{,}4 \cdot 18 = 6{,}24\ \mathrm{s}$$

Then for these values on the basis of formulas mentioned in Tab II were computed values of PI and PID analog and digital controller parameters. The sampling period T = 4 s was estimated from formula (13).

PI analog controller:

$k_P^* = 0{,}18 \qquad T_I^* = 9{,}24\ \mathrm{s}$

PI digital controller:

$k_P^* = 0{,}12 \qquad T_I^* = 7{,}24\ \mathrm{s}$

PID analog controller:

$k_P^* = 0{,}35 \qquad T_I^* = 11{,}76\ \mathrm{s} \quad T_D^* = 2{,}94\ \mathrm{s}$

PID digital controller:

$k_P^* = 0{,}18 \qquad T_I^* = 7{,}76\ \mathrm{s} \quad T_D^* = 1{,}94\ \mathrm{s}$

The control system step responses are shown in Fig. 4. In case of need the controllers can be fine-tuned up by a change of their gains.

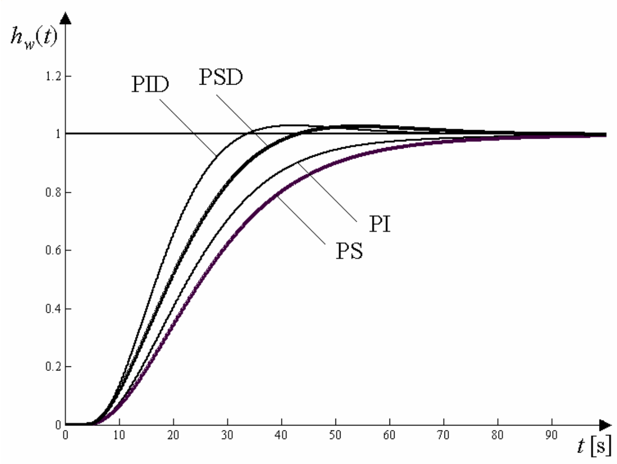

Figure 4: Control system step responses – example

5 CONCLUSION

The described PI and PID analog and digital controller tuning method is easy and effective. It goes up from the desired model method and it easily enables the fine-tuning up by change of only one tuning parameter – the controller gain. The initial value of the varying parameter ensures approximately the marginal non-oscillatory control process. The computational formulas are shown in simple memorizable forms.

The new identification method of the nonoscillatory proportional plants of a higher order and with time delay enables the acquisition of their mathematical models with three parameters in the form of the transfer function of the first or second order with time delay. The important property of the mentioned identification method is that it keeps the equality of the complementary areas over approximated and original plant step responses.

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