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RESIDUAL FILTER DESIGN FOR FAULT DETECTION IN LINEAR DETERMINISTIC DISCRETE-TIME MIMO SYSTEMS

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Abstract

The linear matrix inequality (LMI) based residual generator design approach is presented in this paper. Design conditions are expressed in the terms of LMIs with the matrix rank constraints, implying from Lyapunov equation, which correspond to a feasible solution. Obtained formulation is the convex LMI problem for the full order residual generator design. This handles the optimized structure of the residual generator for the discrete-time linear systems. Given method is demonstrated using a structural system model example, which includes the actuator and sensor fault vectors, respectively.

Keywords: Fault tolerant control, fault diagnosis, residual generators, linear matrix inequalities.

1 INTRODUCTION

The complexity of the control systems requires the fault tolerance schemes to provide control of the faulty system. The fault tolerant systems are such a remarakable applications having potential significance for these domains in which control of the systems must proceed while the system is operative and testing opportunities are limited by operational considerations. The real problem is usually to fix the system with faults so that it can continue its mission over a period of time with certain functionality limitation. The automated diagnosis is one part of these large problems known as fault detection, identification and reconfiguration (FDIR). The practical benefits of an integrated approach to FDIR seem to be considerable, especially when knowledge about fauilure information, available in the fault isolation stage and used in the reconfiguration latency, increases the control reliability and utility. Whereas diagnosis is the problem of identifying elements whose abnormality is sufficient to explain an observed malfunction, reconfiguration can be viewed as the problem of identifying elements whose reconfiguration is sufficient to restore acceptable behavior of the system.

In recent years, significant progress has been achieved in this field where the residual based FDI techniques using the parity spaces, observers and filters schemes, as well as the parameter estimation methods for the residual generator design have been developed [Krokavec and Filasová, 2007]. To obtain robust FDI schemes in presence of disturbances, the optimized residual generators have been proposed.

In this work, the linear matrix inequality (LMI) based H_∞ formulation of the residual generator transfer function design is presented for the discrete-time systems. The fixed-order residual generator is characterized in terms of the convex LMIs, where convex parameterization is represented as a set of LMIs using Lyapunov function. This way, the residual generator can be achieved by the convex optimization for the systems without disturbances. The presented method is a straightforward adaptation of the methodology introduced in [Nobrega et all, 2000] and the integrated procedure provides formulas to use in the residual generator design techniques when designing for the state-space generator structures.

2 ORTHOGONAL COMPLEMENT

The singular value decomposition of a real matrix **X**, $\mathbf{X} \in \mathbf{P}^{hxh}$, with rank(**X**) = $k < h$, gives

$$\mathbf{U}^T\mathbf{X}\mathbf{V} = \begin{bmatrix}\mathbf{U}_1^T \\ \mathbf{U}_2^T\end{bmatrix}\mathbf{X}\begin{bmatrix}\mathbf{V}_1 & \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{\Sigma}_1 & \mathbf{0}_{12} \\ \mathbf{0}_{21} & \mathbf{0}_{22}\end{bmatrix}, \ \mathbf{\Sigma}_1 \in \mathbb{R}^{kxk}, \mathbf{0}_{12} = \mathbf{0}_{21}^T \in \mathbb{R}^{kx(h\text{-}k)}, \mathbf{0}_{22} \in \mathbb{R}^{(h\text{-}k)x(h\text{-}k)} \tag{1}$$

where $\mathbf{U}^T$ is the orthogonal matrix of the left singular vectors and **V** is the orthogonal matrix of the right singular vectors of **X**. Matrix $\mathbf{\Sigma}_1$ is a diagonal positive definite matrix of the form

$$\mathbf{\Sigma}_1 = \begin{bmatrix}\sigma_1 & & \\ & \ddots & \\ & & \sigma_k\end{bmatrix}, \qquad \sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_k > 0 \tag{2}$$

whose diagonal elements are singular values of **X**. Using orthogonal properties of **U**, **V**, i.e.

$$\mathbf{U}^T\mathbf{U} = \mathbf{I}_h, \quad \mathbf{V}^T\mathbf{V} = \mathbf{I}_h, \quad \begin{bmatrix}\mathbf{U}_1^T \\ \mathbf{U}_2^T\end{bmatrix}\begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix} = \begin{bmatrix}\mathbf{I}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2\end{bmatrix}, \quad \mathbf{U}_2^T\mathbf{U}_1 = \mathbf{0} \tag{3}$$

where $\mathbf{I}_{(*)}$ is the identity matrix of the appropriate dimension, then **X** can be written as

$$\mathbf{X} = \mathbf{U}\Sigma\mathbf{V}^T = \begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix}\begin{bmatrix}\mathbf{\Sigma}_1 & \mathbf{0}_{12} \\ \mathbf{0}_{21} & \mathbf{0}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^T \\ \mathbf{V}_2^T\end{bmatrix} = \begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix}\begin{bmatrix}\mathbf{S}_1 \\ \mathbf{0}_2\end{bmatrix} = \mathbf{U}_1\mathbf{S}_1, \quad \mathbf{S}_1 = \mathbf{\Sigma}_1\mathbf{V}_1^T \tag{4}$$

Then (3), (4) implies

$$\mathbf{U}_2^T\mathbf{X} = \mathbf{U}_2^T\mathbf{U}\Sigma\mathbf{V}^T = \mathbf{U}_2^T\begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix}\begin{bmatrix}\mathbf{S}_1 \\ \mathbf{0}_2\end{bmatrix} = \mathbf{0} \tag{5}$$

It is evident that for an arbitrary nonsingular matrix **Y** is

$$\mathbf{X}^{\perp}\mathbf{X} = \mathbf{Y}\mathbf{U}_2^T\mathbf{X} = \mathbf{0}, \quad \mathbf{X}^{\perp} = \mathbf{Y}\mathbf{U}_2^T \tag{6}$$

where the non-unique matrix $\mathbf{X}^{\perp}$ is an orthogonal complement to **X**.

3 SCHUR COMPLEMENT

Let a linear matrix inequality takes the form

$$\begin{bmatrix}\mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{R}\end{bmatrix} > 0, \quad \mathbf{Q} = \mathbf{Q}^T, \quad \mathbf{R} = \mathbf{R}^T \tag{7}$$

Using Gauss elimination yields a consistent result

$$\begin{bmatrix}\mathbf{I} & -\mathbf{S}\mathbf{R}^{-1} \\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0} \\ -\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{I}\end{bmatrix} = \begin{bmatrix}\mathbf{I} & -\mathbf{S}\mathbf{R}^{-1} \\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{Q}-\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{S} \\ \mathbf{0} & \mathbf{R}\end{bmatrix} = \begin{bmatrix}\mathbf{Q}-\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{R}\end{bmatrix} \tag{8}$$

Since

$$\det\begin{bmatrix}\mathbf{I} & -\mathbf{S}\mathbf{R}^{-1} \\ \mathbf{0} & \mathbf{I}\end{bmatrix} = 1 \tag{9}$$

this transform does not change the positivity of (7), i.e. it follows as a consequence

$$\begin{bmatrix}\mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{R}\end{bmatrix} > 0 \quad \Leftrightarrow \quad \begin{bmatrix}\mathbf{Q}-\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{R}\end{bmatrix} > 0 \quad \Leftrightarrow \quad \mathbf{Q}-\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T > 0, \quad \mathbf{R} > 0 \tag{10}$$

respectively.

To solve this problem with the another transform matrix, where

$$\det\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ -\mathbf{S}^T\mathbf{Q}^{-1} & \mathbf{I}\end{bmatrix}=1 \tag{11}$$

it can be obtained

$$\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ -\mathbf{S}^T\mathbf{Q}^{-1} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & \mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\mathbf{Q}^{-1}\mathbf{S}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}=\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ -\mathbf{S}^T\mathbf{Q}^{-1} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{Q} & \mathbf{0}\\ \mathbf{S}^T & \mathbf{R}-\mathbf{S}^T\mathbf{Q}^{-1}\mathbf{S}\end{bmatrix}=\begin{bmatrix}\mathbf{Q} & \mathbf{0}\\ \mathbf{0} & \mathbf{R}-\mathbf{S}^T\mathbf{Q}^{-1}\mathbf{S}\end{bmatrix} \tag{12}$$

$$\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & \mathbf{R}\end{bmatrix}>0 \;\Leftrightarrow\; \begin{bmatrix}\mathbf{Q} & \mathbf{0}\\ \mathbf{0} & \mathbf{R}-\mathbf{S}^T\mathbf{Q}^{-1}\mathbf{S}\end{bmatrix}>0 \;\Leftrightarrow\; \mathbf{R}-\mathbf{S}^T\mathbf{Q}^{-1}\mathbf{S}>0,\ \mathbf{Q}>0 \tag{13}$$

Analogously, for the linear matrix inequality

$$\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & -\mathbf{R}\end{bmatrix}<0,\quad \mathbf{Q}=\mathbf{Q}^T,\quad \mathbf{R}=\mathbf{R}^T \tag{14}$$

it yields

$$\begin{bmatrix}\mathbf{I} & \mathbf{S}\mathbf{R}^{-1}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & -\mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{R}^{-1}\mathbf{S}^T & \mathbf{I}\end{bmatrix}=\begin{bmatrix}\mathbf{I} & \mathbf{S}\mathbf{R}^{-1}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{Q}+\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{S}\\ \mathbf{0} & -\mathbf{R}\end{bmatrix}=\begin{bmatrix}\mathbf{Q}+\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0}\\ \mathbf{0} & -\mathbf{R}\end{bmatrix} \tag{15}$$

$$\det\begin{bmatrix}\mathbf{I} & \mathbf{S}\mathbf{R}^{-1}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}=1 \tag{16}$$

and

$$\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & -\mathbf{R}\end{bmatrix}<0 \;\Leftrightarrow\; \begin{bmatrix}\mathbf{Q}+\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0}\\ \mathbf{0} & -\mathbf{R}\end{bmatrix}<0 \;\Leftrightarrow\; \mathbf{Q}+\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T<0,\ \mathbf{R}>0 \tag{17}$$

respectively.

Following the similar approach to solve for

$$\begin{bmatrix}-\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & \mathbf{R}\end{bmatrix}<0,\quad \mathbf{Q}=\mathbf{Q}^T,\quad \mathbf{R}=\mathbf{R}^T,\quad \det\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{S}^T\mathbf{Q}^{-1} & \mathbf{I}\end{bmatrix}=1 \tag{18}$$

it can be easily shown it becomes

$$\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{S}^T\mathbf{Q}^{-1} & \mathbf{I}\end{bmatrix}\begin{bmatrix}-\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & \mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{Q}^{-1}\mathbf{S}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}=\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{S}^T\mathbf{Q}^{-1} & \mathbf{I}\end{bmatrix}\begin{bmatrix}-\mathbf{Q} & \mathbf{0}\\ \mathbf{S}^T & \mathbf{R}+\mathbf{S}^T\mathbf{Q}^{-1}\mathbf{S}\end{bmatrix}=\begin{bmatrix}-\mathbf{Q} & \mathbf{0}\\ \mathbf{0} & \mathbf{R}+\mathbf{S}^T\mathbf{Q}^{-1}\mathbf{S}\end{bmatrix} \tag{19}$$

$$\begin{bmatrix}-\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & \mathbf{R}\end{bmatrix}<0 \;\Leftrightarrow\; \begin{bmatrix}-\mathbf{Q} & \mathbf{0}\\ \mathbf{0} & \mathbf{R}+\mathbf{S}^T\mathbf{Q}^{-1}\mathbf{S}\end{bmatrix}<0 \;\Leftrightarrow\; \mathbf{R}+\mathbf{S}^T\mathbf{Q}^{-1}\mathbf{S}<0,\ \mathbf{Q}>0 \tag{20}$$

As one can see, these complements offer the possibility to rewrite nonlinear inequalities in the closed matrix LMI form.

4 STABILIZING CONTROL

Let the system under consideration is given by the state-space representation

$$\mathbf{q}(i+1)=\mathbf{F}\mathbf{q}(i)+\mathbf{G}\mathbf{u}(i) \tag{21}$$

$$\mathbf{y}(i)=\mathbf{C}\mathbf{q}(i) \tag{22}$$

with constant matrices $\mathbf{F}\in\mathbf{P}^{nxn}$, $\mathbf{G}\in\mathbf{P}^{nxr}$, and $\mathbf{C}\in\mathbf{P}^{mxn}$.

To formulate the stabilizing control design task it is assumed the matrices **F**, **G** to be real, all system states can be observed or measured, pair (**F**, **G**) is controllable, and the system (21), (22) is controlled by the output feedback control law

$$\mathbf{u}(i)=-\mathbf{K}\mathbf{y}(i)=-\mathbf{K}\mathbf{C}\mathbf{q}(i) \tag{23}$$

where $\mathbf{K} \in \mathbf{P}^{nxm}$ is a constant matrix. Substituting (23) into (21) results in

$$\mathbf{q}(i+1) = (\mathbf{F} - \mathbf{GKC})\mathbf{q}(i) \tag{24}$$

Since the overall system (21) is linear in $\mathbf{q}(i)$, the Lyapunov function $\mathrm{v}(\mathbf{q}(i))$ can be of the form

$$\mathrm{v}(\mathbf{q}(i)) = \mathbf{q}^T(i)\mathbf{P}(i)\mathbf{q}(i) \tag{25}$$

where $\mathrm{v}(\mathbf{q}(i))$ is a quadratic positive definite function of the state of the system, with a symmetric positive definite weighting matrix $\mathbf{P}(i)$. If a steady-state solution of $\mathbf{P}(i)$ exists, evaluating the difference of (25) for the steady-state solution $\mathbf{P}$ of $\mathbf{P}(i)$ gives

$$\Delta\mathrm{v}(\mathbf{q}(i),\mathbf{u}(i)) = \mathbf{q}^T(i+1)\mathbf{P}\mathbf{q}(i+1) - \mathbf{q}^T(i)\mathbf{P}\mathbf{q}(i) < 0 \tag{26}$$

$$\Delta\mathrm{v}(\mathbf{q}(i),\mathbf{u}(i)) = \mathbf{q}^T(i)\Big((\mathbf{F}-\mathbf{GKC})^T\mathbf{P}(\mathbf{F}-\mathbf{GKC}) - \mathbf{P}\Big)\mathbf{q}(i) < 0 \tag{27}$$

$$(\mathbf{F}-\mathbf{GKC})^T\mathbf{P}(\mathbf{F}-\mathbf{GKC}) - \mathbf{P} < 0 \tag{28}$$

respectively.

According to Schur complement property (17), inequality (28) can be rewritten as follows

$$\begin{bmatrix} -\mathbf{P} & -(\mathbf{F}-\mathbf{GKC})^T \\ -(\mathbf{F}-\mathbf{GKC}) & -\mathbf{P}^{-1} \end{bmatrix} < 0, \quad \mathbf{P} > 0 \tag{29}$$

Now, using the composed form inequality (29) implies

$$\begin{bmatrix} \mathbf{0} & (\mathbf{GKC})^T \\ \mathbf{GKC} & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{P} & \mathbf{F}^T \\ \mathbf{F} & \mathbf{P}^{-1} \end{bmatrix} < 0, \quad \mathbf{P} > 0 \tag{30}$$

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{G} \end{bmatrix}\mathbf{K}\begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^T \\ \mathbf{0} \end{bmatrix}\mathbf{K}^T\begin{bmatrix} \mathbf{G}^T & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{P} & \mathbf{F}^T \\ \mathbf{F} & \mathbf{P}^{-1} \end{bmatrix} < 0, \quad \mathbf{P} > 0, \quad \begin{bmatrix} \mathbf{P} & \mathbf{F}^T \\ \mathbf{F} & \mathbf{P}^{-1} \end{bmatrix} > 0 \tag{31}$$

respectively.

Therefore, using (10) inequality (31) can be partioned into the form

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{G} \end{bmatrix}\mathbf{K}\begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^T \\ \mathbf{0} \end{bmatrix}\mathbf{K}^T\begin{bmatrix} \mathbf{0} & \mathbf{G}^T \end{bmatrix} - \begin{bmatrix} \mathbf{P} - \mathbf{F}^T\mathbf{P}\mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{-1} \end{bmatrix} < 0, \quad \mathbf{P} > 0 \tag{32}$$

or analogously, using (13), into the form

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{G} \end{bmatrix}\mathbf{K}\begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^T \\ \mathbf{0} \end{bmatrix}\mathbf{K}^T\begin{bmatrix} \mathbf{0} & \mathbf{G}^T \end{bmatrix} - \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{-1} - \mathbf{F}\mathbf{P}^{-1}\mathbf{F}^T \end{bmatrix} < 0, \quad \mathbf{P} > 0 \tag{33}$$

Denoting

$$\mathbf{G}_{\square}^{\perp} = \begin{bmatrix} \mathbf{0} \\ \mathbf{G} \end{bmatrix}^{\perp} = \begin{bmatrix} \mathbf{0} & \mathbf{G}^{\perp} \end{bmatrix} \tag{34}$$

left-multiplication of (33) by (34) and right-multiplication by its transposition lead to the inequalities

$$-\mathbf{G}_{\square}^{\perp}\begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{-1} - \mathbf{F}\mathbf{P}^{-1}\mathbf{F}^T \end{bmatrix}\mathbf{G}_{\square}^{\perp T} = -\begin{bmatrix} \mathbf{0} & \mathbf{G}^{\perp} \end{bmatrix}\begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{-1} - \mathbf{F}\mathbf{P}^{-1}\mathbf{F}^T \end{bmatrix}\begin{bmatrix} \mathbf{0} \\ \mathbf{G}^{\perp T} \end{bmatrix} < 0, \quad \mathbf{P} > 0 \tag{35}$$

which is equivalent to

$$\mathbf{G}^{\perp}(\mathbf{P}^{-1} - \mathbf{F}\mathbf{P}^{-1}\mathbf{F}^T)\mathbf{G}^{\perp T} > 0, \quad \mathbf{P} > 0 \tag{36}$$

In the same spirit, pre-multiplication of (32) on the left and the right side by the orthogonal complement

$$\mathbf{C}_{\square}^{T\perp} = \begin{bmatrix} \mathbf{C}^T \\ \mathbf{0} \end{bmatrix}^{\perp} = \begin{bmatrix} \mathbf{C}^{T\perp} & \mathbf{0} \end{bmatrix} \tag{37}$$

and its transposition, results in

$$-\mathbf{C}^{T\perp}\begin{bmatrix}\mathbf{P}-\mathbf{F}^T\mathbf{PF} & \mathbf{0}\\ \mathbf{0} & \mathbf{P}^{-1}\end{bmatrix}\mathbf{C}^{T\perp T} = -\begin{bmatrix}\mathbf{C}^{T\perp} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{P}-\mathbf{F}^T\mathbf{PF} & \mathbf{0}\\ \mathbf{0} & \mathbf{P}^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{C}^{T\perp T}\\ \mathbf{0}\end{bmatrix} < 0, \quad \mathbf{P} > 0 \tag{38}$$

which is equivalent to

$$\mathbf{C}^{T\perp}(\mathbf{P}-\mathbf{F}^T\mathbf{PF})\mathbf{C}^{T\perp T} > 0, \quad \mathbf{P} > 0 \tag{39}$$

In this regard, the necessary and sufficient conditions for the existence of **K** satisfying (28) are given by (36) and (39).

The simple way how to obtain any solution satisfying both (36) and (39), is to use in (36) the independent LMI variable **Z**. Therefore, there can be solved the next LMIs

$$\mathbf{C}^{T\perp}(\mathbf{P}-\mathbf{F}^T\mathbf{PF})\mathbf{C}^{T\perp T} > 0, \quad \mathbf{P} > 0 \tag{40}$$

$$\mathbf{G}^{\perp}(\mathbf{Z}-\mathbf{FZF}^T)\mathbf{G}^{\perp T} > 0, \quad \mathbf{Z} > 0 \tag{41}$$

where the matrix variables **P** and **Z** are adjoined with the condition

$$\mathbf{P}-\mathbf{Z}^{-1} > 0, \quad \Rightarrow \quad \begin{bmatrix}\mathbf{P} & \mathbf{I}\\ \mathbf{I} & \mathbf{Z}\end{bmatrix} > 0, \quad \mathbf{Z} > 0 \tag{42}$$

and for computing of the gain matrix **K** is then used that matrix variable from **P**, **Z** (and its inversion), which satisfies as (36) as (39).

5 GAIN MATRIX

Expanding Lyapunov inequality (28) it can be obtained

$$\mathbf{F}^T\mathbf{PF}-\mathbf{C}^T\mathbf{K}^T\mathbf{G}^T\mathbf{PF}-\mathbf{F}^T\mathbf{PGKC}+\mathbf{C}^T\mathbf{K}^T\mathbf{G}^T\mathbf{PGKC} < \mathbf{P} \tag{43}$$

Completing to square in (43) gives

$$(\mathbf{KC}-\mathbf{P}_G\mathbf{G}^T\mathbf{PF})^T\mathbf{P}_G^{-1}(\mathbf{KC}-\mathbf{P}_G\mathbf{G}^T\mathbf{PF})-\mathbf{F}^T\mathbf{PGP}_G\mathbf{G}^T\mathbf{PF} < \mathbf{P}-\mathbf{F}^T\mathbf{PF} \tag{44}$$

$$(\mathbf{KC}-\mathbf{P}_G\mathbf{G}^T\mathbf{PF})^T\mathbf{P}_G^{-1}(\mathbf{KC}-\mathbf{P}_G\mathbf{G}^T\mathbf{PF}) < \mathbf{\Phi}^{-1} \tag{45}$$

respectively, where

$$\mathbf{P}_G^{-1} = \mathbf{G}^T\mathbf{PG} > 0, \quad \mathbf{\Phi}^{-1} = \mathbf{P}-\mathbf{F}^T\mathbf{PF}+\mathbf{F}^T\mathbf{PGP}_G\mathbf{G}^T\mathbf{PF} > 0 \tag{46}$$

Then, by Schur complement formula (45), (46) are equivalent to

$$(\mathbf{KC}-\mathbf{P}_G\mathbf{G}^T\mathbf{PF})\mathbf{\Phi}(\mathbf{KC}-\mathbf{P}_G\mathbf{G}^T\mathbf{PF})^T < \mathbf{P}_G \tag{47}$$

After expanding (47) it yields

$$\mathbf{KC\Phi C}^T\mathbf{K}^T-\mathbf{KC\Phi F}^T\mathbf{PGP}_G-\mathbf{P}_G\mathbf{G}^T\mathbf{PF\Phi C}^T\mathbf{K}^T+\mathbf{P}_G\mathbf{G}^T\mathbf{PF\Phi F}^T\mathbf{PGP}_G < \mathbf{P}_G \tag{48}$$

and completing to another square gives the result

$$\begin{aligned}&(\mathbf{K}^T-\mathbf{\Phi}_C\mathbf{C\Phi F}^T\mathbf{PGP}_G)^T\mathbf{\Phi}_C^{-1}(\mathbf{K}^T-\mathbf{\Phi}_C\mathbf{C\Phi F}^T\mathbf{PGP}_G)+\\&+\mathbf{P}_G\mathbf{G}^T\mathbf{PF\Phi F}^T\mathbf{PGP}_G-\mathbf{P}_G\mathbf{G}^T\mathbf{PF\Phi C}^T\mathbf{\Phi}_C\mathbf{C\Phi F}^T\mathbf{PGP}_G < \mathbf{P}_G\end{aligned} \tag{49}$$

Those, it yields

$$(\mathbf{K}^T-\mathbf{\Phi}_C\mathbf{C\Phi F}^T\mathbf{PGP}_G)^T\mathbf{\Phi}_C^{-1}(\mathbf{K}^T-\mathbf{\Phi}_C\mathbf{C\Phi F}^T\mathbf{PGP}_G) < \mathbf{\Psi} \tag{50}$$

where

$$\mathbf{\Phi}_C^{-1} = \mathbf{C\Phi C}^T > 0, \quad \mathbf{\Psi} = \mathbf{P}_G-\mathbf{P}_G\mathbf{G}^T\mathbf{PF}(\mathbf{\Phi}-\mathbf{\Phi C}^T\mathbf{\Phi}_C\mathbf{C\Phi})\mathbf{F}^T\mathbf{PGP}_G > 0 \tag{51}$$

Since matrix **Ψ** is the positive definite matrix, it can be supposed that there exists an arbitrary matrix **L** such that

$$(\mathbf{K}^T-\mathbf{\Phi}_C\mathbf{C\Phi F}^T\mathbf{PGP}_G)^T\mathbf{\Phi}_C^{-\frac{1}{2}}\mathbf{\Phi}_C^{-\frac{1}{2}}(\mathbf{K}^T-\mathbf{\Phi}_C\mathbf{C\Phi F}^T\mathbf{PGP}_G)=\mathbf{\Psi}^{\frac{1}{2}}\mathbf{L}^T\mathbf{L\Psi}^{\frac{1}{2}},\quad \|\mathbf{L}\|<1 \tag{52}$$

and

$$\mathbf{K}^T=\mathbf{\Phi}_C\mathbf{C\Phi F}^T\mathbf{PGP}_G+\mathbf{\Phi}_C^{\frac{1}{2}}\mathbf{L\Psi}^{\frac{1}{2}} \tag{53}$$

6 RESIDUAL FILTER

The state-space model of the time-invariant linear discrete-time MIMO system with the actuator and the sensor faults is generally described by the equations

$$\mathbf{q}(i+1)=\mathbf{Fq}(i)+\mathbf{G}(\mathbf{u}(i)+\mathbf{f}_a(i)) \tag{54}$$

$$\mathbf{y}(i)=\mathbf{Cq}(i)+\mathbf{Du}(i)+\mathbf{f}_s(i) \tag{55}$$

where **F**, **G**, **C**, and **D** are constant matrices of appropriate dimensions, and $\mathbf{f}_a$, $\mathbf{f}_s$ are vectors of the actuator and sensor faults, respectively.

For this system the objective is to design a stable linear dynamic residual filter with the state-space representation

$$\mathbf{p}(i+1)=\mathbf{Jp}(i)+\mathbf{Ly}(i) \tag{56}$$

$$\mathbf{r}(i)=\mathbf{Vp}(i)+\mathbf{Wy}(i)+\mathbf{Gu}(i) \tag{57}$$

Substituting (55) in (56) and (57) results in

$$\mathbf{p}(i+1)=\mathbf{Jp}(i)+\mathbf{Ly}(i)=\mathbf{Jp}(i)+\mathbf{LCq}(i)+\mathbf{LDu}(i)+\mathbf{Lf}_s(i) \tag{58}$$

$$\mathbf{r}(i)=\mathbf{Vp}(i)+\mathbf{Wy}(i)+\mathbf{Gu}(i)=\mathbf{Vp}(i)+\mathbf{WCq}(i)+(\mathbf{WD}+\mathbf{G})\mathbf{u}(i)+\mathbf{Wf}_s(i) \tag{59}$$

and assembling (54), (58), as well as (59) in the compact form, it can be obtained

$$\begin{bmatrix}\mathbf{q}(i+1)\\ \mathbf{p}(i+1)\end{bmatrix}=\begin{bmatrix}\mathbf{F} & \mathbf{0}\\ \mathbf{LC} & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{q}(i)\\ \mathbf{p}(i)\end{bmatrix}+\begin{bmatrix}\mathbf{G}\\ \mathbf{LD}\end{bmatrix}\mathbf{u}(i)+\begin{bmatrix}\mathbf{G} & \mathbf{0}\\ \mathbf{0} & \mathbf{L}\end{bmatrix}\begin{bmatrix}\mathbf{f}_a(i)\\ \mathbf{f}_s(i)\end{bmatrix} \tag{60}$$

$$\mathbf{r}(i)=\begin{bmatrix}\mathbf{WC} & \mathbf{V}\end{bmatrix}\begin{bmatrix}\mathbf{q}(i)\\ \mathbf{p}(i)\end{bmatrix}+\left(\mathbf{WD}+\mathbf{G}\right)\mathbf{u}(i)+\begin{bmatrix}\mathbf{0} & \mathbf{W}\end{bmatrix}\begin{bmatrix}\mathbf{f}_a(i)\\ \mathbf{f}_s(i)\end{bmatrix} \tag{61}$$

Therefore, the faults free system (60), (61) can now be described as follows

$$\begin{bmatrix}\mathbf{q}(i+1)\\ \mathbf{p}(i+1)\end{bmatrix}=\left(\begin{bmatrix}\mathbf{F} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{W} & \mathbf{V}\\ \mathbf{L} & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{C} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\right)\begin{bmatrix}\mathbf{q}(i)\\ \mathbf{p}(i)\end{bmatrix}+\left(\begin{bmatrix}\mathbf{G}\\ \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{W} & \mathbf{V}\\ \mathbf{L} & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{D}\\ \mathbf{0}\end{bmatrix}\right)\mathbf{u}(i) \tag{62}$$

$$\mathbf{r}(i)=\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{W} & \mathbf{V}\\ \mathbf{L} & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{C} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{q}(i)\\ \mathbf{p}(i)\end{bmatrix}+\left(\mathbf{G}+\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{W} & \mathbf{V}\\ \mathbf{L} & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{D}\\ \mathbf{0}\end{bmatrix}\right)\mathbf{u}(i) \tag{63}$$

Such description can be rewritten in terms of $\mathbf{K}^{\circ}$ in the form

$$\mathbf{q}^{\circ}(i+1)=(\mathbf{F}^{\circ}-\mathbf{G}^{\circ}\mathbf{K}^{\circ}\mathbf{C}^{\circ})\mathbf{q}^{\circ}(i)+(\mathbf{L}^{\circ}-\mathbf{G}^{\circ}\mathbf{K}^{\circ}\mathbf{D}^{\circ})\mathbf{u}(i) \tag{64}$$

$$\mathbf{r}(i)=(\mathbf{M}^{\circ}-\mathbf{H}^{\circ}\mathbf{K}^{\circ}\mathbf{C}^{\circ})\mathbf{q}^{\circ}(i)+(\mathbf{N}^{\circ}-\mathbf{H}^{\circ}\mathbf{K}^{\circ}\mathbf{D}^{\circ})\mathbf{u}(i) \tag{65}$$

where with $\mathbf{q}^{\circ T}(i)=[\mathbf{q}^T(i)\ \mathbf{p}^T(i)]\in P^{2n}$ it is denoted

$$\mathbf{F}^{\circ}=\begin{bmatrix}\mathbf{F} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\in\mathbb{R}^{2n\times 2n},\quad \mathbf{G}^{\circ}=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\in\mathbb{R}^{2n\times 2n},\quad \mathbf{C}^{\circ}=\begin{bmatrix}\mathbf{C} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\in\mathbb{R}^{(m+n)\times 2n} \tag{66}$$

$$\mathbf{K}^{\circ}=-\begin{bmatrix}\mathbf{W} & \mathbf{V}\\ \mathbf{L} & \mathbf{J}\end{bmatrix}\in\mathbb{R}^{2n\times(m+n)},\quad \mathbf{D}^{\circ}=\begin{bmatrix}\mathbf{D}\\ \mathbf{0}\end{bmatrix}\in\mathbb{R}^{2n\times r},\quad \mathbf{H}^{\circ}=\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\in\mathbb{R}^{n\times 2n} \tag{67}$$

$$\mathbf{M}^{\circ}=\mathbf{0}\in\mathbb{R}^{n\times 2n},\quad \mathbf{N}^{\circ}=\mathbf{G}\in\mathbb{R}^{n\times r} \tag{68}$$

Note, the system matrix of such fault residual filter in (64) takes the same structure as the system matrix in (24). Lyapunov inequality for (64) can be equivalently written as

$$(\mathbf{F}^{\circ} - \mathbf{G}^{\circ}\mathbf{K}^{\circ}\mathbf{C}^{\circ})^T \mathbf{P}^{\circ}(\mathbf{F}^{\circ} - \mathbf{G}^{\circ}\mathbf{K}^{\circ}\mathbf{C}^{\circ}) - \mathbf{P}^{\circ} < 0 \tag{69}$$

and (69) can take form (33), i.e.

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{G}^{\circ} \end{bmatrix} \mathbf{K}^{\circ} \begin{bmatrix} \mathbf{C}^{\circ} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^{\circ T} \\ \mathbf{0} \end{bmatrix} \mathbf{K}^{\circ T} \begin{bmatrix} \mathbf{0} & \mathbf{G}^{\circ T} \end{bmatrix} - \begin{bmatrix} \mathbf{P}^{\circ} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{\circ -1} - \mathbf{F}^{\circ}\mathbf{P}^{\circ -1}\mathbf{F}^{\circ T} \end{bmatrix} < 0, \quad \mathbf{P}^{\circ} > 0 \tag{70}$$

and

$$\mathbf{G}_{\square}^{\circ} = \begin{bmatrix} \mathbf{0} \\ \mathbf{G}^{\circ} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \quad \Rightarrow \quad \mathbf{G}_{\square}^{\circ\perp} \square \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{2n} & \mathbf{G}^{\circ\perp} \end{bmatrix} \tag{71}$$

Pre-multiplying (70) from the left side by (71) and from the right side by its inversion, it can be obtained for $\mathbf{P}^{\circ} > 0$

$$\mathbf{G}^{\circ\perp}(\mathbf{P}^{\circ -1} - \mathbf{F}^{\circ}\mathbf{P}^{\circ -1}\mathbf{F}^{\circ T})\mathbf{G}^{\circ\perp T} > 0, \quad \mathbf{P}^{\circ} > 0 \tag{72}$$

Considering that

$$\mathbf{P}^{\circ -1} = \begin{bmatrix} \mathbf{Z} & \mathbf{Z}_{12} \\ \mathbf{Z}_{12}^T & \mathbf{Z}_{22} \end{bmatrix}, \quad \mathbf{G}^{\circ\perp} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{73}$$

it provides for (72)

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \left(\begin{bmatrix} \mathbf{Z} & \mathbf{Z}_{12} \\ \mathbf{Z}_{12}^T & \mathbf{Z}_{22} \end{bmatrix} - \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{Z} & \mathbf{Z}_{12} \\ \mathbf{Z}_{12}^T & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{F}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right) \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} > 0, \quad \mathbf{Z} > 0 \tag{74}$$

which is equivalent to

$$\mathbf{Z} - \mathbf{F}\mathbf{Z}\mathbf{F}^T > 0, \quad \mathbf{Z} > 0 \tag{75}$$

When the structure (32) is utilized, it follows

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{G}^{\circ} \end{bmatrix} \mathbf{K}^{\circ} \begin{bmatrix} \mathbf{C}^{\circ} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^{\circ T} \\ \mathbf{0} \end{bmatrix} \mathbf{K}^{\circ T} \begin{bmatrix} \mathbf{0} & \mathbf{G}^{\circ T} \end{bmatrix} - \begin{bmatrix} \mathbf{P}^{\circ} - \mathbf{F}^{\circ T}\mathbf{P}^{\circ}\mathbf{F}^{\circ} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{\circ -1} \end{bmatrix} < 0, \quad \mathbf{P}^{\circ} > 0 \tag{76}$$

$$\mathbf{C}_{\square}^{\circ T} = \begin{bmatrix} \mathbf{C}^{\circ T} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \Rightarrow \quad \mathbf{C}_{\square}^{\circ T\perp} \square \begin{bmatrix} \mathbf{C}^{\circ T\perp} & \mathbf{0} \end{bmatrix} \tag{77}$$

which equivalent form is

$$\mathbf{C}^{\circ T\perp}(\mathbf{P}^{\circ} - \mathbf{F}^{\circ T}\mathbf{P}^{\circ}\mathbf{F}^{\circ})\mathbf{C}^{\circ T\perp T} > 0, \quad \mathbf{P}^{\circ} > 0 \tag{78}$$

Taking in solution that

$$\mathbf{P}^{\circ} = \begin{bmatrix} \mathbf{Y} & \mathbf{Y}_{12} \\ \mathbf{Y}_{12}^T & \mathbf{Y}_{22} \end{bmatrix}, \quad \mathbf{C}_{\square}^{\circ T\perp} \square \begin{bmatrix} \mathbf{C}^{\circ T\perp} & \mathbf{0} \end{bmatrix} \tag{79}$$

it can be verified, that

$$\begin{bmatrix} \mathbf{C}^{\circ T\perp} & \mathbf{0} \end{bmatrix} \left(\begin{bmatrix} \mathbf{Y} & \mathbf{Y}_{12} \\ \mathbf{Y}_{12}^T & \mathbf{Y}_{22} \end{bmatrix} - \begin{bmatrix} \mathbf{F}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{Y} & \mathbf{Y}_{12} \\ \mathbf{Y}_{12}^T & \mathbf{Y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right) \begin{bmatrix} \mathbf{C}^{\circ T\perp T} \\ \mathbf{0} \end{bmatrix} > 0, \quad \mathbf{Z} > 0 \tag{80}$$

$$\mathbf{C}^{T\perp}(\mathbf{Y} - \mathbf{F}^T\mathbf{Y}\mathbf{F})\mathbf{C}^{T\perp T} > 0, \quad \mathbf{Y} > 0 \tag{81}$$

respectively, i.e. for $\mathbf{Y} = \mathbf{P}$ (81) is equivalent with (78).

7 RESIDUAL FILTER PARAMETER DESIGN

It is evident that Lyapunov function (69) takes the same structure as (28). Therefore, the filter matrices (67) can be derived using the generalized structure of (53) and (46), (51), i.e.

$$\mathbf{K}^{\circ T} = -\begin{bmatrix} \mathbf{W}^T & \mathbf{L}^T \\ \mathbf{V}^T & \mathbf{J}^T \end{bmatrix} = \mathbf{\Phi}_C^\circ \mathbf{C}^\circ \mathbf{\Phi}^\circ \mathbf{F}^{\circ T} \mathbf{P}^\circ \mathbf{G}^\circ \mathbf{P}_G^\circ + \mathbf{\Phi}_C^{\circ \frac{1}{2}} \mathbf{L}^\circ \mathbf{\Psi}^{\circ \frac{1}{2}}, \quad \left\| \mathbf{L}^\circ \right\| < 1 \tag{82}$$

$$\mathbf{P}_G^{\circ -1} = \mathbf{G}^{\circ T} \mathbf{P}^\circ \mathbf{G}^\circ > 0 \tag{83}$$

$$\mathbf{\Phi}^{\circ -1} = \mathbf{P}^\circ - \mathbf{F}^{\circ T} \mathbf{P}^\circ \mathbf{F}^\circ + \mathbf{F}^{\circ T} \mathbf{P}^\circ \mathbf{G}^\circ \mathbf{P}_G^\circ \mathbf{G}^{\circ T} \mathbf{P}^\circ \mathbf{F}^\circ > 0 \tag{84}$$

$$\mathbf{\Phi}_C^{\circ -1} = \mathbf{C}^\circ \mathbf{\Phi}^\circ \mathbf{C}^{\circ T} > 0 \tag{85}$$

$$\mathbf{\Psi}^\circ = \mathbf{P}_G^\circ - \mathbf{P}_G^\circ \mathbf{G}^{\circ T} \mathbf{P}^\circ \mathbf{F}^\circ (\mathbf{\Phi}^\circ - \mathbf{\Phi}^\circ \mathbf{C}^{\circ T} \mathbf{\Phi}_C^\circ \mathbf{C}^\circ \mathbf{\Phi}^\circ) \mathbf{F}^{\circ T} \mathbf{P}^\circ \mathbf{G}^\circ \mathbf{P}_G^\circ > 0 \tag{86}$$

where the design matrices are given in (66) and $\mathbf{L}^\circ$ is a non-square matrix, satisfying (82). In the above, it is implicitly assumed that matrix $\mathbf{P}^\circ$ is known, i.e. $\mathbf{P}^\circ$ is a solution of (69).

Partitioning $\mathbf{P}^\circ$ and $\mathbf{P}^{\circ -1}$, with the same notation as in (73) and (79), gives

$$\mathbf{P}^\circ = \begin{bmatrix} \mathbf{Y} & \mathbf{Y}_{12} \\ \mathbf{Y}_{12}^T & \mathbf{Y}_{22} \end{bmatrix}, \quad \mathbf{P}^{\circ -1} = \begin{bmatrix} \mathbf{Z} & \mathbf{Z}_{12} \\ \mathbf{Z}_{12}^T & \mathbf{Z}_{22} \end{bmatrix}, \quad \mathbf{P}^\circ \mathbf{P}^{\circ -1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{87}$$

$$\mathbf{I} = \mathbf{Y}\mathbf{Z} + \mathbf{Y}_{12}\mathbf{Z}_{12}^T, \quad \mathbf{Z}_{12}^T = -\mathbf{Y}_{22}^{-1}\mathbf{Y}_{12}^T\mathbf{Z} \tag{88}$$

$$\mathbf{I} = (\mathbf{Y} - \mathbf{Y}_{12}\mathbf{Y}_{22}^{-1}\mathbf{Y}_{12}^T)\mathbf{Z} \tag{89}$$

respectively, and using Sherman – Morrison – Woodbury equality

$$(\mathbf{A} + \mathbf{B}\mathbf{C}\mathbf{D})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{D}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{D}\mathbf{A}^{-1} \tag{90}$$

it can be obtained

$$\mathbf{Z} = (\mathbf{Y} - \mathbf{Y}_{12}\mathbf{Y}_{22}^{-1}\mathbf{Y}_{12}^T)^{-1} = \mathbf{Y}^{-1} - \mathbf{Y}^{-1}\mathbf{Y}_{12}(-\mathbf{Y}_{22} + \mathbf{Y}_{12}^T\mathbf{Y}^{-1}\mathbf{Y}_{12})^{-1}\mathbf{Y}_{12}^T\mathbf{Y}^{-1} \tag{91}$$

$$\mathbf{Y}(\mathbf{Z} - \mathbf{Y}^{-1})\mathbf{Y} = \mathbf{Y}_{12}(\mathbf{Y}_{22} - \mathbf{Y}_{12}^T\mathbf{Y}^{-1}\mathbf{Y}_{12})^{-1}\mathbf{Y}_{12}^T \tag{92}$$

Since (87) also implies

$$\mathbf{I} = \mathbf{Y}_{12}^T\mathbf{Z}_{12} + \mathbf{Y}_{22}\mathbf{Z}_{22}, \quad \mathbf{Z}_{12} = -\mathbf{Y}^{-1}\mathbf{Y}_{12}\mathbf{Z}_{22} \tag{93}$$

$$\mathbf{Z}_{22}^{-1} = \mathbf{Y}_{22} + \mathbf{Y}_{12}^T\mathbf{Z}_{12}\mathbf{Z}_{22}^{-1} = \mathbf{Y}_{22} - \mathbf{Y}_{12}^T\mathbf{Y}^{-1}\mathbf{Y}_{12}\mathbf{Z}_{22}\mathbf{Z}_{22}^{-1} = \mathbf{Y}_{22} - \mathbf{Y}_{12}^T\mathbf{Y}^{-1}\mathbf{Y}_{12} \tag{94}$$

substituting (94) in (92) finally results in

$$\mathbf{E} = \mathbf{Y}\mathbf{Z}\mathbf{Y} - \mathbf{I} = \mathbf{Y}_{12}\mathbf{Z}_{22}\mathbf{Y}_{12}^T \tag{95}$$

Hence, using SVD property

$$\mathbf{E} = \mathbf{Y}\mathbf{Z}\mathbf{Y} - \mathbf{I}, \quad \mathbf{E} = \mathbf{V}_E\mathbf{\Sigma}_E\mathbf{T}_E^T, \quad \mathbf{V}_E = \left[\mathbf{v}_{E1}\ \mathbf{v}_{E2} \cdots \mathbf{v}_{En}\right], \quad \mathbf{\Sigma}_E = \mathrm{diag}\left[\sigma_{E1}\ \sigma_{E2} \cdots \sigma_{En}\right] \tag{96}$$

where $\mathbf{\Sigma}_E$ is the matrix of singular values of $\mathbf{E}$ and $\mathbf{V}_E$ is the matrix of the associated singular vectors, it can be chosen

$$\mathbf{Y}_{12} = \mathbf{V}_E, \quad \mathbf{Z}_{22} = \mathbf{\Sigma}_E \quad \Rightarrow \quad \mathbf{Y}_{22} = \mathbf{Z}_{22}^{-1} + \mathbf{Y}_{12}^T\mathbf{Y}^{-1}\mathbf{Y}_{12} \tag{97}$$

to complete the elements of $\mathbf{P}^\circ$.

Matrix $\mathbf{P}^\circ$, which elements are computed using (96), (97), and $\mathbf{Y}$, $\mathbf{Z}$ satisfies (81), (75), respectively, can be used to compute the residual filter parameter from (82) – (86) instead of any another matrix $\mathbf{P}^\circ$, possibly obtained directly from (69).

8 ILLUSTRATIVE EXAMPLE

To demonstrate the algorithm properties it was assumed that system is given by (54) - (55), where

$$\mathbf{F} = \begin{bmatrix} 0.9993 & 0.0987 & 0.0042 \\ -0.0212 & 0.9612 & 0.7775 \\ -0.3875 & -0.7187 & 0.5737 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 0.0051 & 0.0050 \\ 0.1029 & 0.0987 \\ 0.0387 & -0.0388 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \mathbf{D} = \mathbf{0}$$

Applying Matlab function *svd*(.), the orthogonal complements were obtained as

$$\mathbf{C}^{T\perp} = \begin{bmatrix} -0.5774 & -0.5774 & 0.5774 \end{bmatrix}, \quad \mathbf{G}^{\perp} = \begin{bmatrix} -0.9987 & 0.0500 & -0.0014 \end{bmatrix}$$

Using Self-Dual-Minimization (SeDuMi) package for Matlab the output-feedback gain matrix problem was solved as feasible with the matrices

$$\mathbf{P} = \begin{bmatrix} 7.3196 & 2.1052 & 2.5690 \\ 2.1052 & 7.5368 & 1.8613 \\ 2.5690 & 1.8613 & 2.8825 \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} 3.8739 & -0.0587 & -0.1002 \\ -0.0587 & 4.7617 & -0.0555 \\ -0.1002 & -0.0555 & 4.9905 \end{bmatrix}$$

where only **P** satisfies (36) and (39). For that **P,** upon some computation, it was found

$$\mathbf{P}_G^{-1} = \begin{bmatrix} 0.1024 & 0.0742 \\ 0.0742 & 0.0648 \end{bmatrix}, \quad \mathbf{\Phi}^{-1} = \begin{bmatrix} 2.3562 & 1.8594 & 2.5633 \\ 1.8594 & 7.5246 & 1.8610 \\ 2.5633 & 1.8610 & 2.8825 \end{bmatrix}, \quad \mathbf{\Phi}_C^{-1} = \begin{bmatrix} 0.6599 & -1.3804 \\ -1.3804 & 11.9435 \end{bmatrix}$$

and with L = 0 the final results were

$$\mathbf{K}^T = \begin{bmatrix} 7.0353 & -8.5785 \\ 1.7392 & -1.7844 \end{bmatrix}, \quad \mathbf{F}_c = \mathbf{F} - \mathbf{GKC} = \begin{bmatrix} 1.0063 & 0.0988 & 0.0113 \\ 0.1016 & 0.9584 & 0.1974 \\ -0.9926 & -0.8582 & -0.1680 \end{bmatrix}$$

Closed-loop eigenvalues spectrum is $\rho(\mathbf{F}_c) = \{0.0000, 0.8471, 0.9496\}$.

Solving the above for residual filter parameter, using (75) and (81) it was obtained

$$\mathbf{Y} = \begin{bmatrix} 1.1986 & 0.3208 & 0.3976 \\ 0.3208 & 1.6333 & 0.3203 \\ 0.3976 & 0.3203 & 0.5207 \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} 0.4069 & -0.1546 & 0.1554 \\ -0.1546 & 0.3685 & 0.2065 \\ 0.1554 & 0.2065 & 0.6645 \end{bmatrix}$$

Substituting the above parameter matrix into (95), and applying SVD to matrix **E** gives

$$\mathbf{E} = \begin{bmatrix} -0.1904 & 0.3506 & 0.4727 \\ 0.3506 & 0.1792 & 0.4819 \\ 0.4727 & 0.4819 & -0.6238 \end{bmatrix}$$

$$\mathbf{\Sigma}_E = \mathbf{Z}_{22} = \begin{bmatrix} 0.9804 & 0.3208 & 0.3976 \\ 0.3208 & 0.7176 & 0.3203 \\ 0.3976 & 0.3203 & 0.3722 \end{bmatrix}, \quad \mathbf{V}_E = \mathbf{Y}_{12} = \begin{bmatrix} -0.4190 & 0.5146 & -0.7481 \\ -0.2375 & 0.7331 & 0.6373 \\ 0.8764 & 0.4447 & -0.1849 \end{bmatrix}$$

respectively, and

$$\mathbf{Y}_{22} = \begin{bmatrix} 4.1459 & 0.3230 & 0.0426 \\ 0.3230 & 1.9459 & -0.0270 \\ 0.0426 & -0.0270 & 3.6148 \end{bmatrix}, \quad \mathbf{Y}_{22}^{-1} = \begin{bmatrix} 0.2444 & -0.0406 & -0.0032 \\ -0.0406 & 0.5207 & 0.0044 \\ -0.0032 & 0.0044 & 0.2767 \end{bmatrix}$$

Now, providing a construction for matrix (83), it is evident that for $\mathbf{C}^{\circ}$ given in (66) matrix (83) is a singular matrix. Therefore, using pseudo-inversion of (83), i.e.

$$\mathbf{P}_G^{\circ-1} = \mathbf{G}^{\circ T}\mathbf{P}^{\circ}\mathbf{G}^{\circ} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}_{22} \end{bmatrix}, \quad \mathbf{P}_G^{\circ} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}_{22}^{-1} \end{bmatrix}$$

it is obvious, that the generalized gain matrix $\mathbf{K}^{\circ T}$ takes degenerative structure

$$\mathbf{K}^{\circ T} = \begin{bmatrix} 0 & 0 & 0 & -0.0400 & -0.0581 & 0.1958 \\ 0 & 0 & 0 & -0.0764 & -0.0790 & -0.1787 \\ 0 & 0 & 0 & 0.5441 & -0.3225 & -0.0206 \\ 0 & 0 & 0 & -0.2850 & 0.1690 & 0.0108 \\ 0 & 0 & 0 & -0.0263 & 0.0156 & 0.0010 \end{bmatrix}$$

where matrix parameters $\mathbf{J}$ and $\mathbf{L}$ are

$$\mathbf{L} = \begin{bmatrix} -0.0400 & -0.0764 \\ -0.0581 & -0.0790 \\ 0.1958 & -0.1787 \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} 0.5441 & -0.2850 & -0.0263 \\ -0.3225 & 0.1690 & 0.0156 \\ -0.0206 & 0.0108 & 0.0010 \end{bmatrix}, \quad \mathrm{eig}(\mathbf{J}) = \begin{bmatrix} 0.7140 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

and $\mathbf{V}$ and $\mathbf{W}$ are free parameters to abolish linear dependency from residual filter. Since all eigenvalues of $\mathbf{J}$ lie in unit circle, residual filter is stabile.

9 CONCLUDING REMARKS

The residual filter parameter design method is presented, as the generalization of the output feedback control design method. The method uses the standard LMI numerical optimization procedures to manipulate the residual generator matrices as LMI variables. This way, the eigenvalues of the residual generator system matrix $\mathbf{J}$ are placed in the unit circle in the z-plane and the input matrix $\mathbf{L}$ is designed in consequence. The rest filter matrix parameters are free to move out the linear dependency from the residual filter output.

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