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## THE ROBUST, ACCELERATION LOOP SPEED SERVOS

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### Abstract

This paper deals with the problematics of acceleration loop speed servos with a subordinate acceleration controller. The acceleration signal is generated by the analog speed sensor, using the averager and differentiator. The using an acceleration loop in speed (and position) control circuit we are able to achieve better qualities of such circuit. This solution has properties of robust system. The solution is suitable to meet the requirements for a wide range of rpm regulation, run uniformity as well as the devices featuring either non-stationary or extremely high load inertia moment.

**Keywords:** speed servo, the robust servo, the parametric invariant system, the acceleration sensor, the speed control, the angular velocity sensor

### 1 Introduction

The speed servos, in which a speed-voltage generator is used as an angular velocity sensor, either have a subordinate current-loop or in case a current-limited supply unit servodrive is used, such a loop is absent. Such a circuit together with an appropriate controller (usually of PI type) and converter comprise a set with satisfactory qualities in terms of the speed control and the load-moment invariance.

### 2 Theory

We can see a simplified example of the structure of mentioned circuit in Fig.1, where the dc servodrive speed control with the PI controller is depicted. Symbols :  $w$  – servodrive shaft angular velocity,  $w_0$  – desired speed,  $e_w$  – speed error,  $M_m$  – servodrive torque,  $K_1$  – controller (and converter) proportional amplification,  $K_2$  – controller (and converter) integration amplification,  $R$  – servodrive armature circuit resistance,  $C$  – servodrive torque coefficient,  $J$  – servodrive (and load) inertia moment,  $M_z$  – load moment,  $U_m$  – term voltage of electric servomotor,  $I$  – current of servomotor,  $s$  – Laplace operator.

The electromagnetic time constant and the effects of friction and saturation are all neglected.

The transfer functions of individual input for a servo with such structure are as follows :  
- the control transfer function

$${}^1F_w(s) = \frac{w(s)}{w_0(s)} = \frac{\frac{K_1}{K_2} s + 1}{\frac{JR}{K_2 C} s^2 + \frac{C + K}{K_2} s + 1} \quad (1)$$

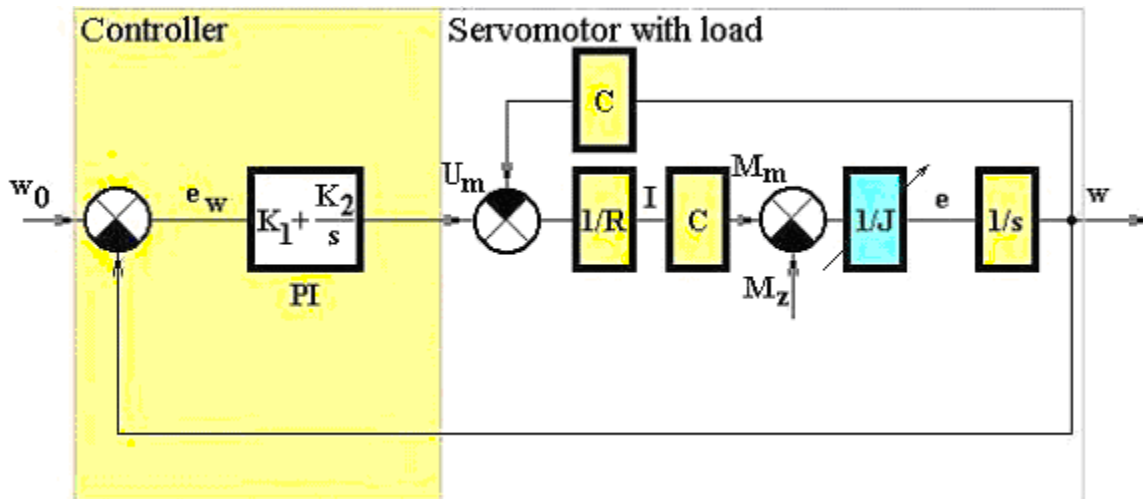


Fig. 1 Speed control circuit

- the error transfer function

$${}^1F_U(s) = \frac{w(s)}{M_z(s)} = \frac{\frac{R}{K C} s}{\frac{JR}{K_2 C} s^2 + \frac{C + K_1}{K_2} s + 1} \tag{2}$$

It follows from the aforementioned formulas that the control transfer functions and invariance with respect to the load moment are both mainly dependant of the amplification magnitude of  $K_2$  controller integration part, where the value of this amplification is finite and its magnitude in respect of the stability is limited. Thus the possibilities for further improvement of mentioned qualities are depleted. When considering an improvement of the function of speed controller we may conclude that it is possible to further improve the qualities of the system in Fig.1 on the condition that the angular velocity control loop (i.e. acceleration loop) is integrated into the circuit.

In our first approach let's assume that reliable analog angular velocity sensor is at our disposal and under these circumstances we may examine the qualities of the speed controller with subordinate acceleration loop.

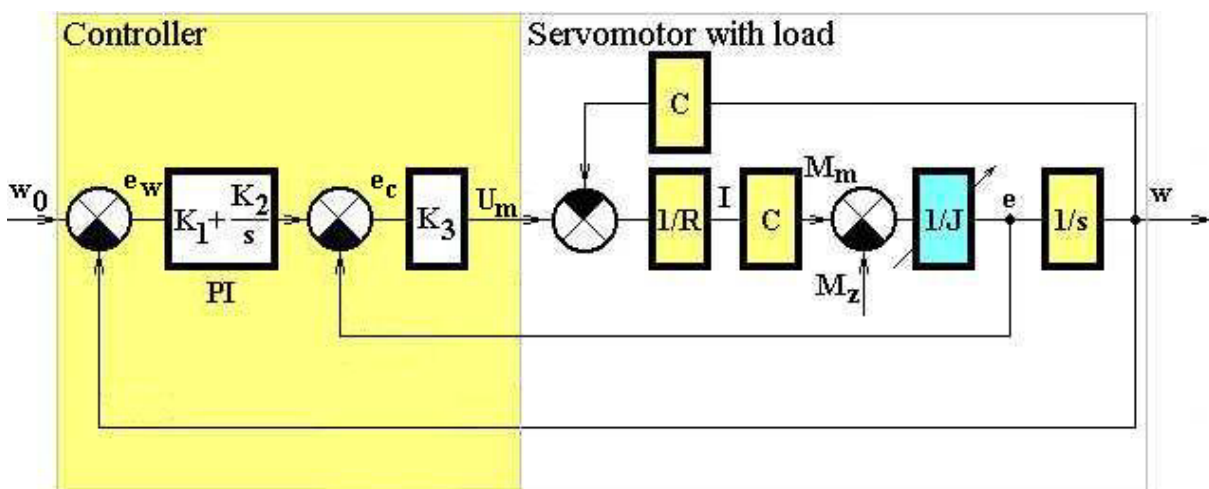


Fig. 2 Acceleration loop speed control circuit

Simplified block diagram of this circuit is illustrated in Fig.2, where the denomination of quantities remains the same except for additional ones :  $e_c$  – acceleration error,  $e$  – servodrive shaft angular velocity,  $K_3$  – accelerator controller and actuator (converter) amplification.

Assume the PI speed controller and the proportional acceleration controller. The control transfer function for this structure of the control circuit is :

$${}^2F_w(s) = \frac{w(s)}{w_0(s)} = \frac{\frac{K_1}{K_2}s + 1}{\left(\frac{RJ}{K_2K_3C} + \frac{1}{K_2}\right)s^2 + \left(\frac{C}{K_2K_3} + \frac{K_1}{K_2}\right)s + 1} \quad (3)$$

and the error transfer function of the system with acceleration loop :

$${}^2F_U(s) = \frac{w(s)}{M_z(s)} = \frac{\frac{R}{K_2K_3C}s}{\left(\frac{RJ}{K_2K_3C} + \frac{1}{K_2}\right)s^2 + \left(\frac{C}{K_2K_3} + \frac{K_1}{K_2}\right)s + 1} \quad (4)$$

If those two formulas are compared, one can see that we are dealing with the systems with the same type of transfer function. Having compared the corresponding terms in numerator and denominator we may conclude the following implications of the use of acceleration loop :

- a) if we assume the same pair of amplifications ( $K_1, K_2$ ) and the following applies to  $K_3$  :

$$K_3 > 1 \quad (5)$$

then there is no change in zeros of the numerator of control transfer function. It follows from the error transfer functions that by applying the acceleration loop, the invariance with respect to the error moment rises, because :

$$\frac{R}{K_2K_3C} < \frac{R}{K_2C} \quad (6)$$

- b) complying with the assumptions of a) article, it can be inferred using the denominator quadratic terms coefficients comparison that there is very little change (or increase) in system damping

$$\frac{C}{K_2K_3} + \frac{K_1}{K_2} \geq \frac{K_1}{K_2} \quad (7)$$

for  $K_2K_3$  is substantially large and  $C$  constant is small.

The coefficient corresponding with the time constant square in case of the system in Fig.2.

$$\frac{RJ}{K_2K_3C} + \frac{1}{K_2} \approx \frac{1}{K_2} \quad (8)$$

We are considering a large inertia moment applied to the servodrive shaft, as this corresponds with the servodrive operating mode in robots and also other devices where :

$$J = J_m + (10 \div 30) J_m \quad (9)$$

where  $J_m$  is the servodrive inertia moment. We assume a large value of  $K_2K_3$  at the same time. In such case, changes of inertia moment have virtually no effect on the magnitude of time constant. On the account of the fact that the inertia moment is not found in formulas 3 and 4, we may consider this circuit to be invariant with respect to the changes of inertia moment, i.e. parametrically invariant, thus a robust system.

The pros of the acceleration loop speed controller are clearly evident, though the source of reliable acceleration signal remains a problem. This issue can be addressed by using a circuit that samples the signal from the speed sensor (speed-voltage generator TG), subtracts the values of consecutive speed samples thus determining the difference proportional to the angular acceleration, instead of a continuous acceleration signal. The solution of such system is depicted in Fig.3. The speed-voltage generator voltage ( $U_{ws}(t_2)$ ) proportional to the angular velocity ( $w(t_2)$ ) is fed to the averager, generating a speed-voltage generator signal bereft of parasitic components at its output. This signal is sampled by a sampler with  $T$  time period and fed to the zero-order holding circuit (ZOHC), resulting in a transformation of the continuous signal to the staircase analog speed signal  $U_w^*$ . This signal is fed to the differentiator  $D$  at moments determined by the logic control circuit (LCC), and there the speed signal value from the previous time moment ( $t_1; t_1 < t_2$ ) stored in the analog memory  $AMI$  is subtracted from it. The difference of signals pertaining to acceleration is fed to the analog memory  $AM2$ , the task of which is to maintain a signal proportional to the acceleration until it is changed again at the moment  $t_3$ . This process is being repeated periodically and we get the staircase analog acceleration signal fit to the input of acceleration controller at the output of analog memory  $AM2$ .

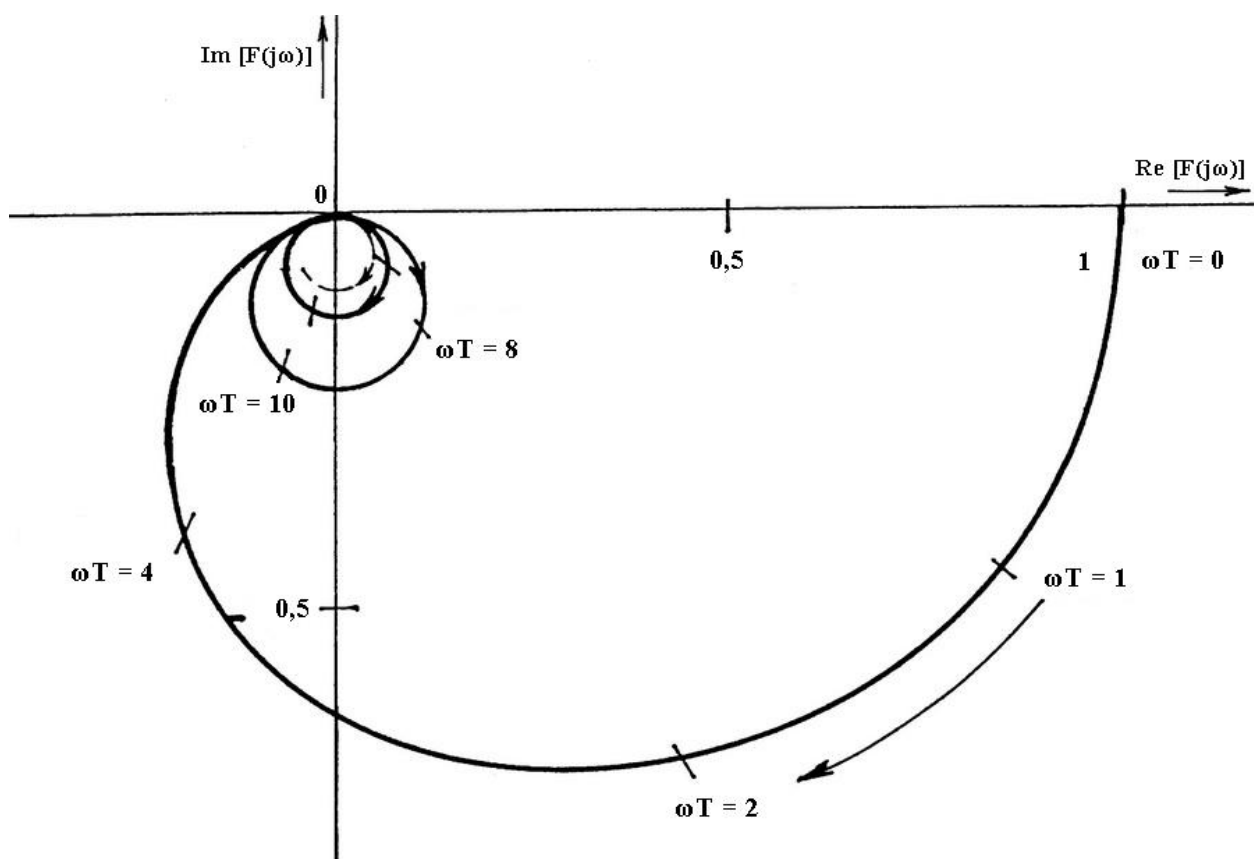


Fig. 3 Frequency characteristics of the averager

The transfer function of the averager in frequency domain is :

$$F(j\omega) = \frac{2}{\omega T} \sin\left(\frac{\omega T}{2}\right) e^{-j\frac{\omega T}{2}} \quad (10)$$

If  $\omega T = \pm 2\pi, \pm 4\pi, \pm 6\pi$  and so on, we have a phase change of  $\pm \pi, \pm 2\pi, \pm 3\pi$  and so on, however the transfer function of amplitude is zero. The course of averager frequency characteristics is depicted in Fig 4. We may conclude that the averager implemented in the feedback loop of control system doesn't reduce the degree of stability at critical frequencies.

Before proceeding to another considerations concerning the averager performance with respect to Fig. 4 (i.e. the controlled integrator with sampler and the zero-order holding circuit) it is necessary to introduce a notion of continual averager. By this notion we mean such an element, at the output of which we get the continual value of average input signal. The output values equal the input values of averager at the moments of sampling with  $T$  sampling period.

Unlike the averager, which is physically feasible, it is necessary to think of the continual averager as a mathematical fiction, while not physically feasible, yet very useful when judging the averager performance and realistically describing its performance at the moments of sampling. It is possible to observe that the continual averager with sampler and holding circuit has the same performance as averager.

The image transfer function of continual averager can be expressed in the following form :

$$F(s) = \frac{1 - e^{-sT}}{sT} \quad (11)$$

One may check the correctness of this statement in ([1],[2]). The choice of sampling period  $T$  is based on a comparison of servodrive time constants with respect to the Shannon-Kotelnikov theorem. For the HSM60 servodrive, used as an example in the next chapters, we assume  $T = 1,4$  ms (700 Hz).

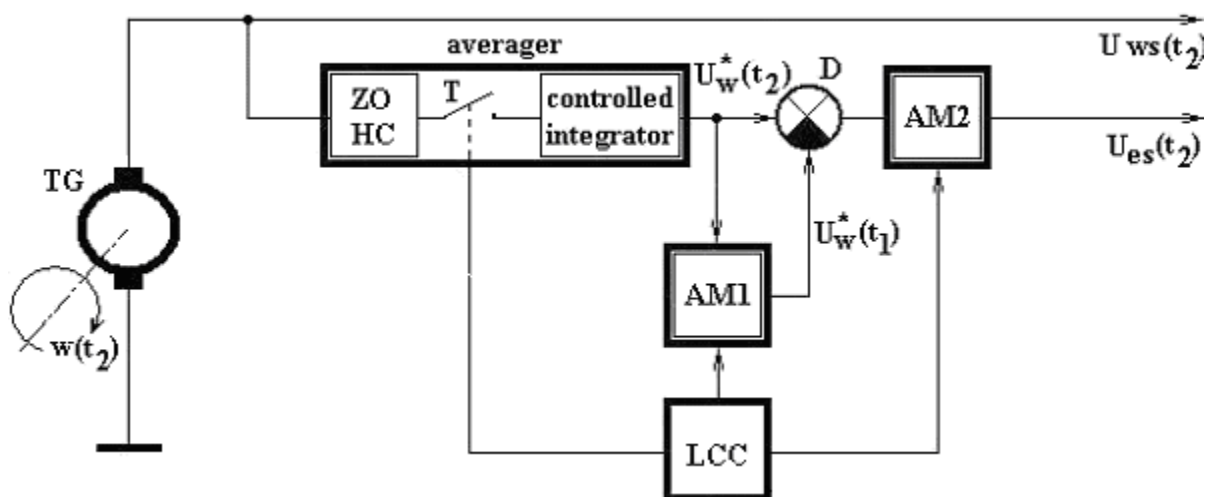


Fig. 4 Schematics of angular acceleration sensor

### 3 Implementation of Acceleration Loop Control Circuit

If a sampling period is chosen appropriately, we may consider a staircase analog signal equal to a continuous signal with regard to the practical use. In such case, the acceleration loop speed

control circuit (ROR-A) is feasible. The block diagram is in Fig.5. It is the electric servomotor type HSM60 servodrive speed control circuit with the K4A5 tachogenerator ( $K_W = 0,0184$  Vs/rad). The acceleration sensor is considered a proportional unit with amplification of  $K_e = 0,876$  Vs<sup>2</sup>/rad and converter amplification of  $K_M = 0,7$ . The controllers were set to the following values :  $K_1 = 7,02$ ;  $K_2 = 452,7$ ;  $K_3 = 14,7$ .

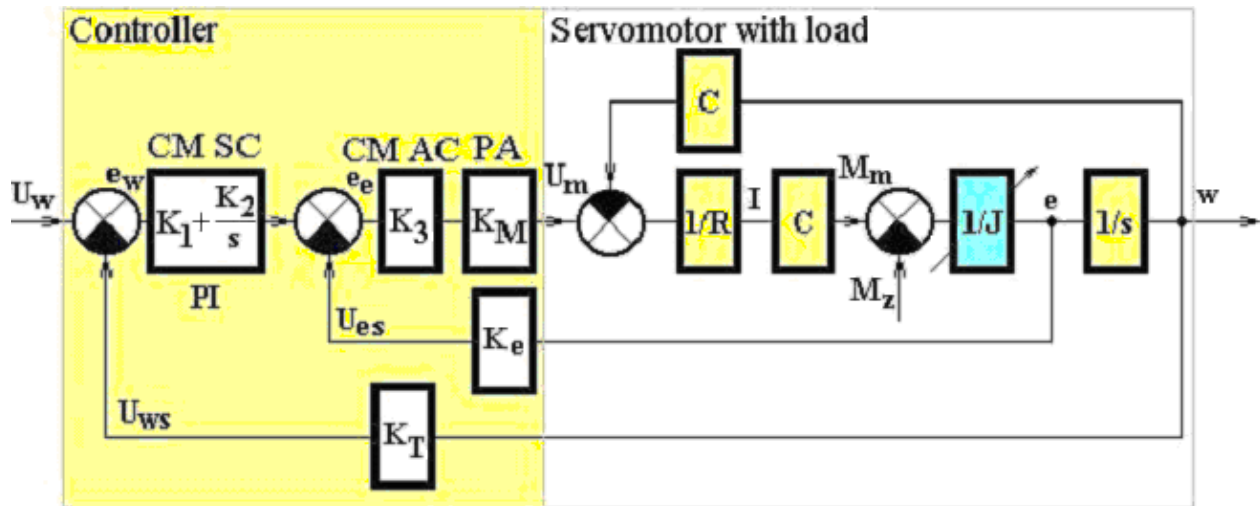


Fig. 5 Structure of acceleration loop speed control circuit



Fig. 6 Implementation of a speed servosystem

Simultaneously, the same control circuit albeit without the acceleration loop (i.e. without the acceleration sensor ( $K_e = 0$ ) and acceleration controller ( $K_3 = 1$ )) had been implemented. The remaining values of amplification were the same. The following formulas stand for the control transfer function and error transfer function of the circuit in Fig.5 respectively :

$${}^3F_w(s) = \frac{w_s(s)}{U_w(s)} = \frac{\frac{1}{K_T} \left( \frac{K_1}{K_2} s + 1 \right)}{\frac{1}{K_T} \left( \frac{RJ}{K_2 K_3 K_M C} + \frac{K_e}{K_2} \right) s^2 + \left( \frac{C}{K_2 K_3 K_M K_T} + \frac{K_1}{K_2} \right) s + 1} \quad (12)$$

$${}^3F_U(s) = \frac{w_s(s)}{M_z(s)} = \frac{\frac{R}{K_2 K_3 K_M K_T C} s}{\frac{1}{K_T} \left( \frac{RJ}{K_2 K_3 K_M C} + \frac{K_e}{K_2} \right) s^2 + \left( \frac{C}{K_2 K_3 K_M K_T} + \frac{K_1}{K_2} \right) s + 1} \quad (13)$$

## 4 Experimental Part

Both control circuits were tested at the indicated amplification values and given parameters of used devices. The transient responses were measured with varying magnitude of dynamic load, i.e. using flies 10 to 75 times the inertia moment of the servodrive. The transient responses of the speed control circuit with PI controller (ROR) are shown in Fig.7. Fig.8 shows the transient responses of the speed control circuit with acceleration loop (ROR-A). As is obvious from the responses, those at the load 10 to 30 times the servodrive inertia moment are close to each other and compared to Fig.7 we see that, the circuit is virtually robust in this range. When overloaded (that is at the load 75 times the inertia moment), the acceleration loop is capable of providing a regulation process in a reasonable time and at incomparably higher quality compared to a common control circuit. In Fig.10, the transient response together with the appropriate measured acceleration course is illustrated. Due to the proportional relationship between C and J quantities (C/J), the course of servodrive current I is similar to the course of acceleration, what is clearly visible in Fig.5.

One of the distinct advantages of acceleration loop speed control circuit is its steady-state run uniformity. The responses of servodrive run found out experimentally when the effects of friction are most evident are shown in Fig.9. These responses were recorded as a tachogenerator voltage. One can see that the introduction of acceleration loop helps to reduce the run non-uniformity to one-fourth its original value. The acceleration loop circuit also allows for an increase of speed regulation, in our case from 1:5000 to 1:30000. These regulation ranges were determined by the maximum servodrive rpm (5000 rpm) related to the lowest attainable. The maximum rpm were measured in a form of tachogenerator voltage while the lowest ones were evaluated from the time needed to complete one revolution of the servodrive shaft.

## 5 Conclusion

The key feature of the robust servosystem solution is a tendency to control such a quantity, that is closest to the source of parametric errors as well as the input of error quantities in the block diagram of the system. Under the closeness we mean such a position of the signal of controlled quantity where the number of astatic elements between this quantity and an error input is minimal. In case of angular velocity control it is astaticism of first order. Thus the implementation of angular acceleration control circuit is an optimal solution. In such case, there is no astatic element between the source of parametric error (total inertia moment) and other



errors (friction moment) (Fig.2 and 5). It is possible to apply said method of system robustification to other types of control system as well (for example hydraulic servodrives, pneumatic artificial muscles etc.)

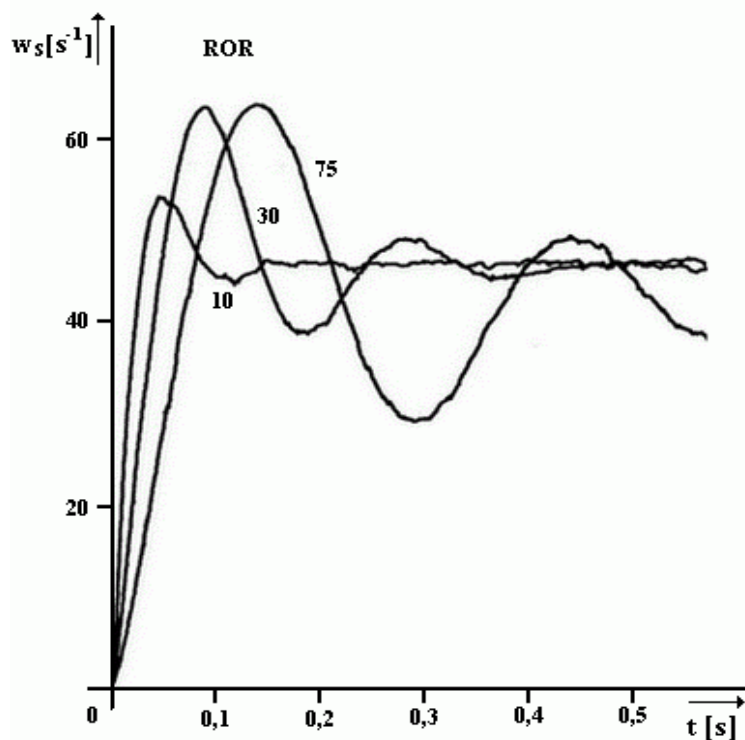


Fig. 7 Transient responses of the speed control circuit

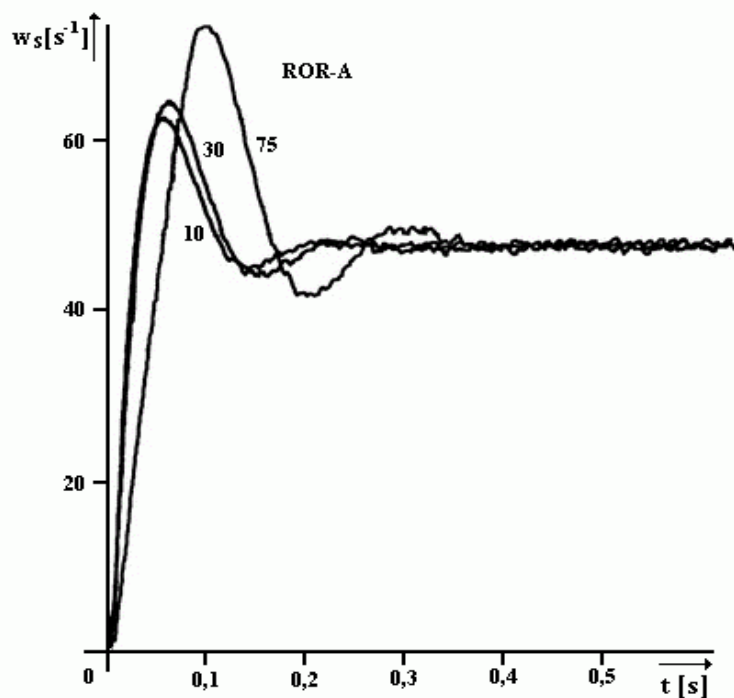


Fig. 8 Transient responses of the speed control circuit with the acceleration loop

To conclude, by using an acceleration loop in speed control circuit we are able to achieve better qualities of such circuit while keeping the complexity and cost of necessary technical equipment reasonably low. This solution is suitable to meet the requirements for a wide range of rpm regulation, run uniformity as well as the devices featuring either non-stationary or extremely high load inertia moment.

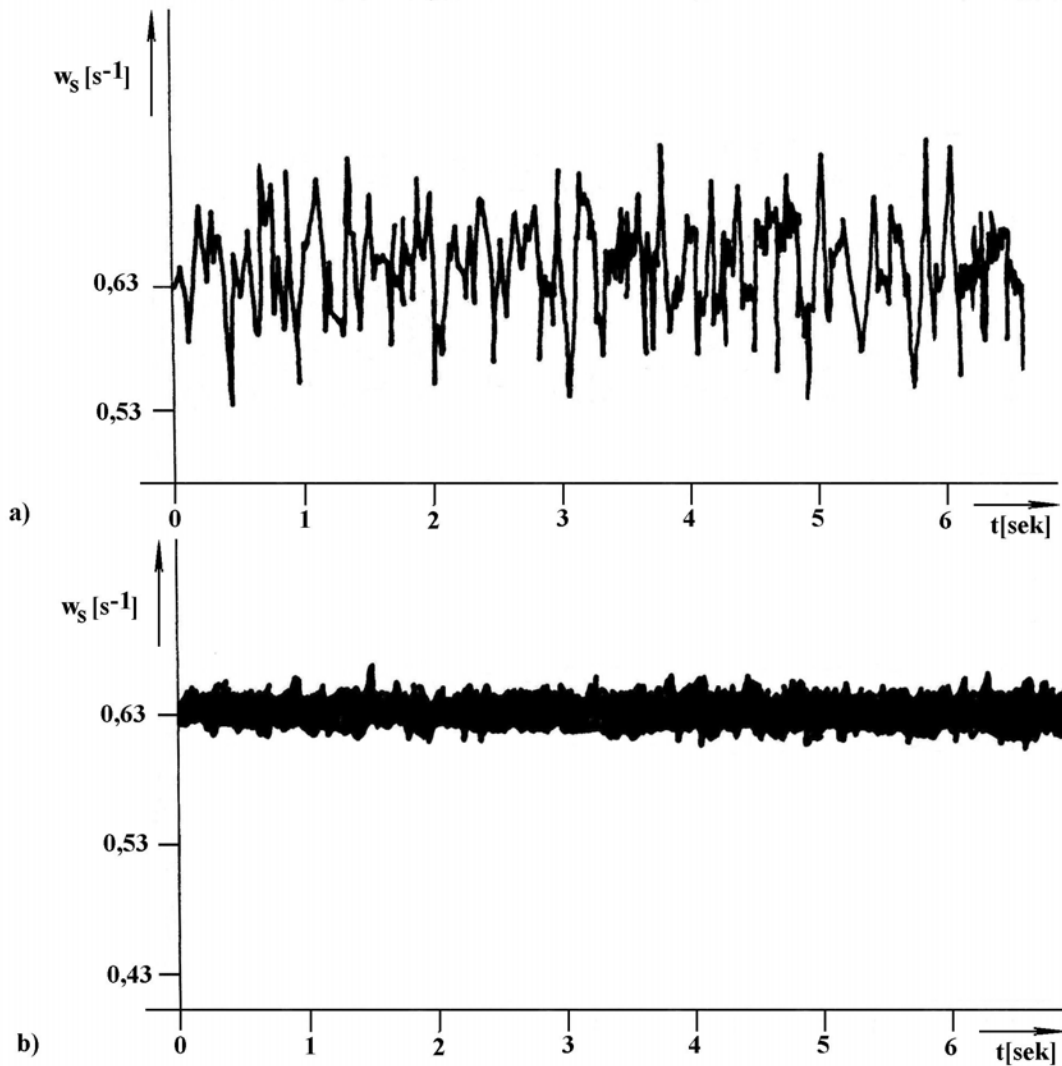


Fig. 9 Evenness of turning of a common servosystem (a) and an acceleration loop servosystem (b)

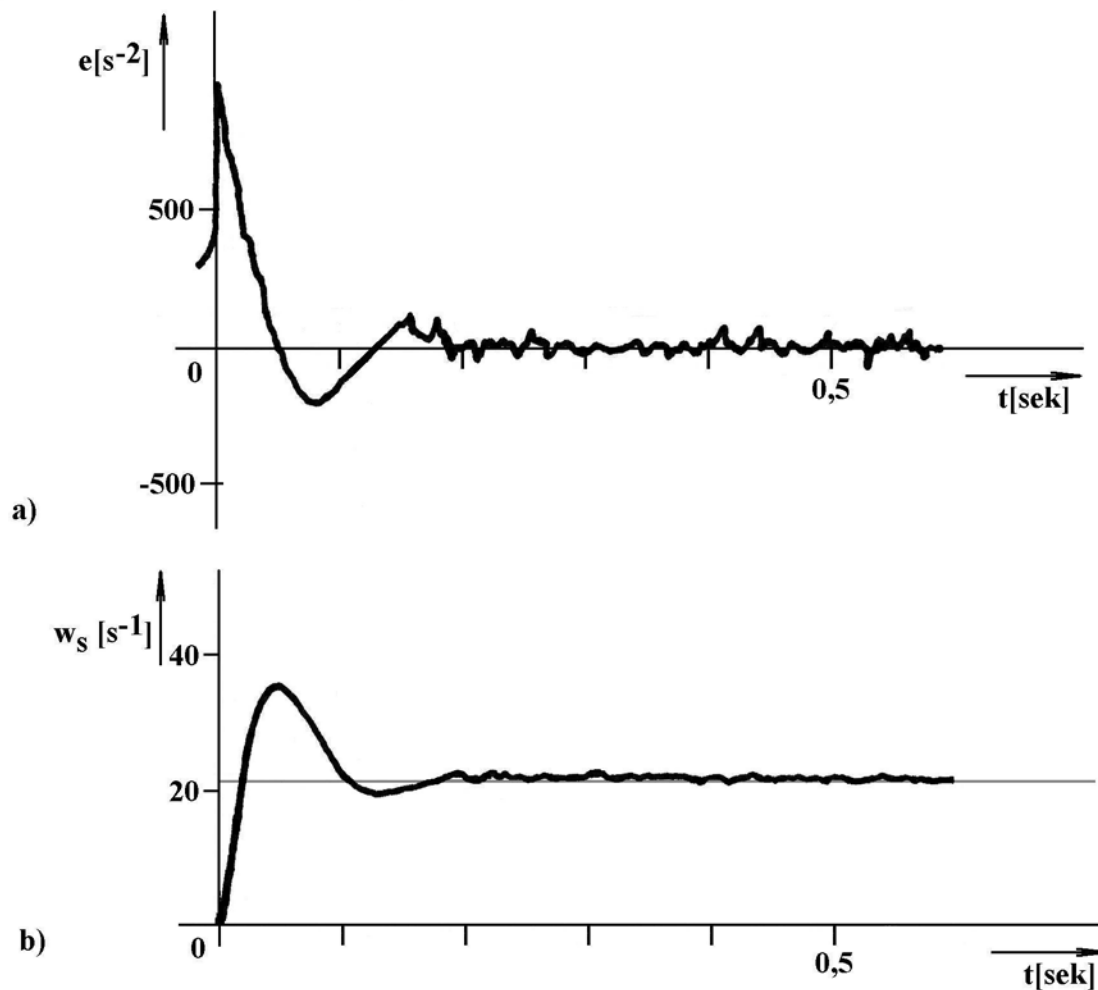


Fig. 10 Course of speed and acceleration at  $J=20J_m$

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