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MULTICRITERIAL SYNTHESIS OF LOGISTIC SYSTEMS THROUGH THE HIERARCHY ANALYSIS

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1 Abstract

The existing methods of multicriterial synthesis of logistics systems (LS) have been developed and improved according to the complex of technical and economic indications. Capabilities of procedures of scale evaluation and evaluation of the coordination of expert opinion have been expanded, what increased the efficiency of problems solutions on LS synthesis with the quantity and quality criteria.

Keywords: Logistics System; Multicriterial Logistics System; Logistics System Synthesis; : Logistics System Synthesis; Multicriterial Synthesis.

1 INTRODUCTION

In the modern scientific literature and entrepreneur activity, several definitions of logistics are used. The American Engineer – Logistics Society defines logistics as the art and science on rational planning, control and management of movement of material and informational flows in space and time and from their primary source to the final consumer [1, 2]. In the entrepreneur activity, logistics means control of material flows at the enterprise: from reception of raw materials until the supply of ready products to a consumer. Designers of computer systems and managers explain logistic as mathematical and software instruments for LS engineering and reengineering.

We consider logistic system hereinafter as a complex organizational and technological system, which consists of the complex of interconnected material and information flows, which are joined in internal and external targets into one management process. Logistic systems are created with the purpose to receive synergic effect, which significantly increases the amount of particular effects from each of these flows at their individual performance, at the expense of increase of connections between financial, production, transportation and commercial flows.

2 PROBLEM FORMULATION: MULTICRITERIAL LS SYNTHESIS

At the initial stages of LS design, it is envisioned to use simplified methods of multicriterial synthesis of the organizational structure and component-by-component composition of the created system. When there are quantity and quality target functions, the project decision making faces up significant difficulties, caused by weak formalization of the solved problem under presence of significant ambiguity. The first stage towards the solution of the considered problem is the construction of a mathematical model of the LS synthesis problem.

The model of the problem of a multicriterial LS synthesis can be represented as a complex of target functions $f_i, i = \overline{1, m}$ and a set of alternatives of the made decisions $x = \{x_j\} \subset X, j = \overline{1, n}$ in the form [3-6]:

$$f_i(X) \rightarrow \max_{x \in X}, \quad i = \overline{1, m}, \quad j = \overline{1, n}$$

where m is the number of target functions,

$x = \{X_1, X_2, \dots, X_n\} \subset X$ is a final multitude of alternatives of the made decisions, which contains n elements of x_j . The value of the numbers m and n should be relatively small, as they determine the calculation complexity of dialog procedures in the real time scale on reception of additional information on the problem.

To determine the most preferable alternatives of the made decision considering all particular criteria, let's use the method of linear convolution.

$$J(x_j) = \sum_{i=1}^m \alpha_i f_i(x_j), \quad j = \overline{1, n}.$$

If there is no ground to consider the multitude of achievability of the considered multicriterial problem prominent, then instead of the linear convolution it is expedient to use a generalized Joffrion criteria based on the combination of the linear and maximize convolution.

Structurizing the problem on LS multicriterial synthesis includes the decomposition of the initial task to simpler components and the construction of the multilevel graphic vision in the form of hierarchical structure of the problem of decision-making. Building of a hierarchical structure starts with setting the global target at the scheme (hierarchy focus), under which there is a hierarchical structure of local criteria containing the target levels, sub targets and target functions. Below the hierarchical structure of local criteria there is a hierarchical structure of local alternatives of the made decisions.

To determine the relative importance of objects in the hierarchical structure, we use a scale of preferences, which allows an expert to assign numbers to the objects to evaluate them. These numbers α_{ij} show how many times object p_i is more preferred to an object p_j . The minimal value of numbers α_{ij} in the scale of relations is limited by the complexity of the calculation procedures.

According to the results of expert opinions of decision-makers, relative significance of particular criteria and alternatives of the made decisions regarding particular criteria, which are on various levels of the hierarchy is determined. Relative significance is expressed in numbers in the form of vectors of priorities, which represent the so-called strict evaluations in the scale of relations.

To solve problems on multicriterial LS synthesis, specialized methods of optimization [3-6] are used which allow finding the decisions under conditions of weak structurizing of the created systems and noncoordination of the initial information, given by the set of quality and quantity decisions. Well-grounding and reliability of the made decisions are expressed through properties of coherency and transitiveness between the expert evaluations of the initial factor space. Breach of these properties results in ambiguous choice on the multitude of criteria and alternatives of the made decisions.

3 THE METHOD OF THE HIERARCHY ANALYSIS

The analysis of the hierarchy is in principle based on the decomposition of the complex problem through a complex of more simple components of the hierarchy, described by the author of this method T.Saaty [7]. The problems are divided into the objects-criteria and objects-alternatives of the made decisions. Based on objects-criteria, a hierarchical structure of criteria is organized, which contains levels of targets and sub targets; based on objects-alternatives, a hierarchical structure of alternatives of made decisions is created. In the result of decomposition, the structure of the problem is organized, reflecting relative degree of interconnections of the objects of the hierarchy.

The solution of a problem for multicriteria LS synthesis through the method of hierarchy analysis includes the following operations:

1. Setting the meaning of the problem for multicriterial project problems.
2. Mathematical setting of a problem to make multicriterial project decisions, which includes forming the hierarchical structure of alternative connections on the made decisions.

3. Ranging the final multitude of objects–criteria and objects–alternatives on the made decisions $p = \{p_1, \dots, p_i, \dots, p_m\}$ according to the importance, through setting weight ratio vectors $\alpha = \{\alpha_1, \dots, \alpha_i, \dots, \alpha_m\}$ satisfying the constraints:

$$\sum_{i=1}^m \alpha_i = 1, \alpha_i \geq 0$$

In the hierarchical system of decision–making, ranging on importance of each k level of the multitude of objects–criteria and objects–alternatives, $p^k = \{P_1^k, \dots, P_i^k, \dots, P_m^k\}$ is carried out through setting k vectors of weight ratios

$$\alpha^k = \{\alpha_1^k, \dots, \alpha_i^k, \dots, \alpha_{m_k}^k, \dots\}, \quad k = \overline{1, K},$$

where k is the number of levels of the hierarchy of objects–criteria and objects–alternatives; m_k is the number of objects on the k level of criteria and alternatives of the made decisions.

The problem on objects ranging within the limits of each k level of the hierarchy includes the following: based on the survey of experts and methods to process expert data, to determine the multitude of relationship $P_i^k \rightarrow \alpha_i^k$ for all the levels of hierarchical structure of criteria and hierarchical structure of alternatives.

4. Formation of matrices of pair comparisons $[S_p^k \text{ } mxm]$ for k level of objects

$$[S_p^k \text{ } mxm] = \begin{matrix} p_1^k & \left| \begin{array}{cccc} p_1^k & \dots & p_i^k & \dots & p_m^k \\ \alpha_{11}^k & \dots & \alpha_{ij}^k = \alpha_1^k / \alpha_j^k & \dots & \alpha_{1m}^k \\ \dots & \dots & \dots & \dots & \dots \\ p_i^k & \left| \begin{array}{cccc} \alpha_{i1}^k & \dots & \alpha_{ij}^k = \alpha_i^k / \alpha_j^k & \dots & \alpha_{im}^k \\ \dots & \dots & \dots & \dots & \dots \\ p_m^k & \left| \begin{array}{cccc} \alpha_{m1}^k & \dots & \alpha_{mj}^k = \alpha_m^k / \alpha_j^k & \dots & \alpha_{mm}^k \end{array} \right. \end{array} \right. \end{matrix}$$

Matrices of pair comparisons $[S_p^k \text{ } mxm]$ are main store places for information, necessary to make multicriterial decisions. Every matrix $[S_p^k \text{ } mxm]$ is created according to the following rules:

Opinion of each expert (user of LPR) is written as a line of a matrix of a pair comparison $[S_p^k \text{ } mxm]$.

An expert should be experienced in the field of made decisions and able to answer quickly the given questions: how many times the weight ratio α_i is bigger than the weight ratio α_j or how many times the weight ratio α_j is less than the ratio α_i .

Each element α_{ij}^k of the matrix of pair comparisons $[S_p^k m \times m]$ is determined by the expression $\alpha_{ij}^k = \alpha_i^k / \alpha_j^k$, where α_i^k and α_j^k are the weight ratios of the priority of objects of pair connection of k level of the hierarchy of objects-criteria and objects-alternatives:

$$P_i^k \rightarrow \alpha_i^k, P_j^k \rightarrow \alpha_j^k, \alpha_i^k, \alpha_j^k, \rightarrow \alpha_{ij}^k = \alpha_i^k / \alpha_j^k$$

The dimension of the paired ratios α_i^k and α_j^k should be the same, and the value of these ratios cannot be divided by zero. If $\alpha_i^k / \alpha_j^k > 1$, then the object p_i is considered more important than the object p_j . Values of weight ratios received in this way are the evaluations in the scale of relationship and correspond to the so-called strict evaluations.

Checking of coordination of expert opinions is an outcoming prerequisite of the MAI. To determine the dimension of coordination of expert solutions, an initial matrix of pair comparisons is used, which is received based on the survey of experts by the method of pair comparisons according to Saaty [7] scale. As a dimension of coordination, the index of coordination and coordination relationship are often used [5, 7]. The coordination of the reversely symmetrical initial matrix of pair comparisons is equivalent to demand the equality of maximal eigenvalues λ_{\max} to the number of the compared objects n, that is $\lambda_{\max} = n$.

4 LS ANALYSIS EXAMPLE

Below, we consider an example of multicriteria LS synthesis, where it is necessary to choose a preferable variant of system and software means with the consideration of the following complex of technical and economic indications: f1(x)-system flexibility; f2(x)-system productivity, f3(x) –unit platform; f4(x) –reliability of the unit and software of the system: f5(x)- economic efficiency of a definite variant (alternative A1, A2, A3, A4, A5) of the system, given in table 1.

At the first stage of the procedure of the synthesis, the hierarchical structure of alternatives of the made decisions is designed. Building of the synthesis hierarchy starts with the setting of the hierarchy focus, where the global criteria are placed. Next, there are levels of particular criteria and subcriteria, and below there are levels of alternatives of made decisions (Fig. 1).

Table 1. Alternative of variants of the LS instrumental means

Particular Criteria	Alternative Variants of LS Instrumental Means				
	A1	A2	A3	A4	A5
F1(x)	Easy adaptation	Comparatively flexible	Truly Flexible	Comparatively high	Comparatively high
F2(x)	Up to 2 mil subscribers	Up to 1 mil subscribers	Without limits	500 0 500000 subscribers	Without limits
F3(x)	Operation Systems Windows NT, Linux	OS Windows NT, UNIX	OS Windows NT	Windows NT	Windows NT, UNIX
F4(x)	Very high	Comparatively high	High reliability	High reliability	Comparatively high
F5(x)	High	Not high	Rather high	Not high	Comparatively high

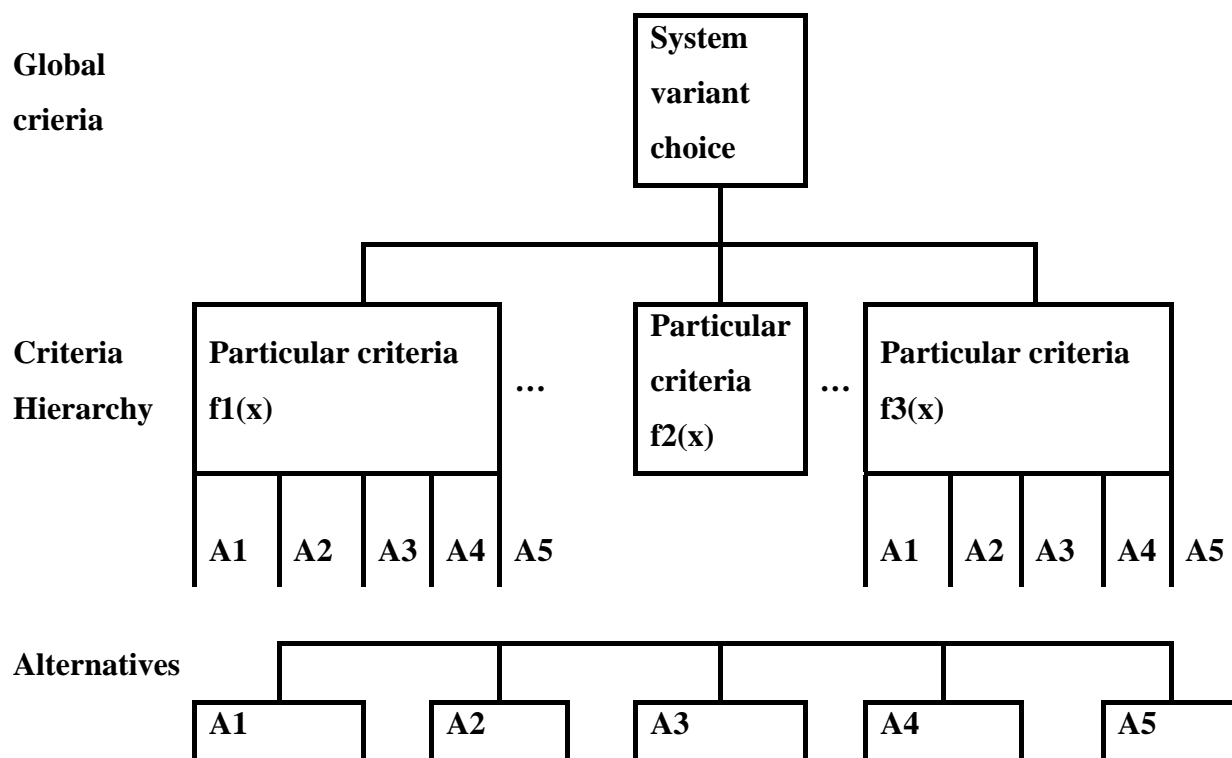


Fig. 1. Hierarchical structure of the LS Synthesis Problem

At the second stage of the synthesis procedure, a matrix of pair comparisons is formed $[S_{f_{mxm}}]$, the elements of which represent the relationship of the superiority of particular criteria to each

other. Diagonal elements of the matrix $[S_{f5X5}]$ can be equal to zero; they are determined by the results of expert surveys with further calculation of all the lacking ratios according to formula:

$$\alpha_{ij} = 1/\alpha_{ji}, \quad \alpha_{ij} = \alpha_i/\alpha_j, \quad \alpha_{ij} = \alpha_{ik} \times \alpha_{kj}, \quad \frac{\alpha_i}{\alpha_k} \cdot \frac{\alpha_k}{\alpha_j} = \frac{\alpha_i}{\alpha_j}$$

that is why a measure of noncoordination is usually considered to be a normed deviation of λ_{\max}

from n , which is called a noncoordination index:
$$ИC = \frac{\lambda_{\max} - n}{n - 1}$$

To evaluate the degree of coordination of expert opinions, the coordination index (ИC) is compared to a random index СИ. A random index is a coordination index, calculated for square n -measure positive reversely symmetric matrix, elements of which are generated by the generator of random numbers for the interval from 1 to 9. In Table 1, there is an average square value of the coordination for random matrix from 1 to 10.

Table 2 . Dimension of random coordination

Matrix dimension	1	2	3	4	5	6	7	8	9	10
Random coordination	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

On receiving the noncoordination index and on choosing a random index from Table 2 for the set matrix order, the coordination relationship is calculated (OC):

$$OC = ИC / СИ$$

if the dimension $OC \leq 0.1$ then the degree of coordination of expert data is considered acceptable. In the opposite way (if $OC > 0.1$), the expert is recommended to reconsider his/her judgments, which introduce the maximal input to the dimension of coordination relationship to the less point based on deeper analysis of the problem.

Based on the results of expert surveys, an initial matrix $[S_{f5X5}]$ of pair comparisons of particular criteria is formed:

$$[S_{f5X5}] = \begin{array}{c} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{array} \begin{array}{ccccc} f_1 & f_2 & F_3 & F_4 & f_5 \\ \left| \begin{array}{ccccc} 1 & 1/3 & 4 & 1 & 2 \\ 3 & 1 & 3 & 1 & 3 \\ 1/4 & 1/3 & 1 & 1/5 & 1/3 \\ 1 & 1 & 5 & 1 & 5 \\ 1/4 & 1/3 & 3 & 1/5 & 1 \end{array} \right| \end{array}$$

and the normed matrix $[N_{f_{5 \times 5}}]$ of pair comparison of particular criteria:

	f_1	f_2	f_3	f_4	f_5	V_{f_i}
F1	0.174	0.111	0.260	0.294	0.177	0.201
F2	0.522	0.334	0.187	0.294	0.265	0.321
F3	0.043	0.111	0.063	0.059	0.029	0.061
F4	0.174	0.334	0.313	0.294	0.441	0.311
F5	0.087	0.110	0.187	0.059	0.088	0.106

Elements $v_{f_{ij}}$ of the normed matrix $[N_{f_{5 \times 5}}]$ are calculated with the use of elements $\alpha_{f_{ij}}$ of the initial matrix $[S_{f_{5 \times 5}}]$ according to the formula:

$$v_{f_{ij}} = \alpha_{f_{ij}} / \sum_{i=1}^m \alpha_{f_{ij}}; j = \overline{1, m}.$$

For example, the element $v_{f_{11}}$ of the first line of the first row $[N_{f_{5 \times 5}}]$ is:

$$\begin{aligned} v_{f_{11}} &= \alpha_{f_{11}} / (\alpha_{f_{11}} + \alpha_{f_{21}} + \alpha_{f_{31}} + \alpha_{f_{41}} + \alpha_{f_{51}}) = \\ &= 1 / (1 + 3 + 1/4 + 1 + 1/4) = 0.174 \end{aligned}$$

Relative values of the weight ratios $V_{f_1} - V_{f_5}$ of particular criteria $f_1(X) - f_5(X)$ are calculated as average values of elements of the corresponding lines of the normed matrix $[N_{f_{5 \times 5}}]$ based on the following formula:

$$V_{f_i} = \frac{1}{m} \sum_{j=1}^m v_{f_{ij}}; i = \overline{1, m}$$

For example, the weight ratio V_{f_1} of the particular criteria $f_1(X)$ is:

$$\begin{aligned} V_{f_1} &= (v_{f_{11}} + v_{f_{12}} + v_{f_{13}} + v_{f_{14}} + v_{f_{15}}) / m = \\ &= (0.174 + 0.111 + 0.260 + 0.294 + 0.177) / 5 = 0.201 \end{aligned}$$

The determination of the weight ratios is carried out according to the following scheme as the definition of ratios of the superiority of particular criteria. For each particular criterion (f_1 -flexibility, f_2 -productivity, f_3 -unit platform, f_4 -reliability, f_5 -economy), the initial $[S_{f_i \ n \times n}]$ and normed $[N_{f_i \ n \times n}]$ matrices are built:

	A_1	A_2	A_3	A_4	A_5
A1	1	4	3	6	3
A2	1/4	1	2	5	2
A3	1/3	1/2	1	7	1
A4	1/6	1/5	1/7	1	1/7
A5	1/3	1/2	1	7	1

		A ₁	A ₂	A ₃	A ₄	A ₅	V _{f2 Aj}	
[N _{f2 5x5}]	=	A1	0.482	0.645	0.419	0.231	0.419	0.439
	A2	0.12	0.161	0.279	0.192	0.279	0.206	
	A3	0.159	0.081	0.139	0.269	0.139	0.157	
	A4	0.079	0.032	0.020	0.038	0.020	0.037	
	A5	0.159	0.081	0.139	0.269	0.139	0.157	

Values of weight ratios of the productivity criteria, given in the line $V_{f2 Aj}$ of the matrix $[N_{f2 5x5}]$ shows that the preferable alternatives on criteria of productivity are the variant A1 with the weight ratio $V_{f2 A1} = 0.439$ and variant A2 with weight ratio $V_{f2 A2} = 0.206$. The value of the ratio of the relative coordination of the initial matrix $[S_{f2 5x5}]$ equals to 8.15%, what is less than the margin allowed: 10% - beyond which it is necessary to reconsider the judgments of experts or to change their composition.

To evaluate the reliability factor, the initial matrix of pair comparisons is build $[S_{f4 5x5}]$ and normed $[N_{f4 5x5}]$. Values of the weight ratios of the reliability criteria prove that the preferred alternatives on reliability criteria are the variant A1 with the weight $V_{f4 A1}=0.448$, variant A2 with ratio $V_{f4 A2}=0.211$ and the variant A5 with the ratio $V_{f4 A5}=0.211$. The value of the relative coordination of the matrix $[S_{f4 5x5}]$ is equal to 3.59%, what is less than the border dimension of 10%, which does not allow considering that the opinions of experts are coordinated. Factors of flexibility, unit platform and economy of the system are calculated in the analogous way.

5 CONCLUSION

The results of the carried out research on LS synthesis show that as the preferred variant of multicriterial synthesis the alternative A1 can be considered - the system with the maximal value of the global priority, in spite of its high cost. If a client does not have enough funds to purchase the alternative A1, then the choice of a cheaper variant of the system is carried out through recalculation of all the tables including the additional information on new demands on the developed project and the coordination of expert opinions. The developed method of multicriterial synthesis can be used when reengineering of large-scale corporative systems, optimization of telecommunication systems, design of instruments on training and retraining of employees, as well as in modeling the process on knowledge control in the systems of distance learning.

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